THE ANNUAL MEETING OF THE SOCIETY

The forty-fourth Annual Meeting of the American Mathematical Society was held at Butler University, Indianapolis, Indiana, from Tuesday to Thursday, December 28–30, 1937, in conjunction with the meetings of the American Association for the Advancement of Science, the Mathematical Association of America, the Association for Symbolic Logic, and the Institute of Mathematical Statistics.

There were four general and eight sectional sessions of the Society, at which three addresses and ninety-three research papers (fifty-nine in person) were given. All sessions were in Jordan Hall, Butler University, except the Gibbs Lecture, which was held in the ballroom of the Antlers Hotel.

Arrangements for the meetings were made by a committee of which Professor K. P. Williams was chairman and Professors Juna L. Beal and B. C. Getchell were the local members. Through the generosity of Butler University, rooms were available in the nearby fraternity and sorority houses. The Marott Hotel was used as the headquarters for mathematicians.

The membership attendance was the second largest at an Annual Meeting, the record being two hundred eighty-two in 1928 at New York City. More than four hundred persons registered, among whom were the following two hundred seventy-one members of the Society:

V. W. Adkisson, R. P. Agnew, L. V. Ahlfors, A. A. Albert, E. S. Allen, C. B. Allendoerfer, Warren Ambrose, V. A. Ames, N. L. Anderson, H. F. Archibald, R. C. Archibald, Emil Artin, Max Astrachan, J. V. Atanasoff, C. S. Atchison, W. L. Ayres, R. W. Babcock, H. W. Bailey, E. F. Beckenbach, F. A. Beeler, M. M. Beenken, A. A. Bennett, William Betz, G. D. Birkhoff, G. A. Bliss, Henry Blumberg, L. M. Blumenthal, O. K. Bower, M. G. Boyce, J. W. Bradshaw, R. W. Brink, H. W. Brinkmann, F. L. Brooks, G. S. Bruton, H. E. Buchanan, C. T. Bumer, R. S. Burington, L. E. Bush, Jewell H. Bushey, W. E. Byrne, W. D. Cairns, B. H. Camp, E. J. Camp, H. H. Campaigne, M. E. Carlen, W. E. Cederberg, J. H. Chanler, Alonzo Church, E. H. Clarke, M. D. Clement, L. M. Coffin, L. W. Cohen, J. B. Coleman, E. G. H. Comfort, N. B. Conkwright, A. H. Copeland, Max Coral, J. J. Corliss, A. P. Cowgill, L. C. Cox, A. T. Craig, H. B. Curry, D. R. Curtiss, J. H. Curtiss, E. H. Cutler, D. R. Davis, H. T. Davis, W. M. Davis, D. B. DeLury, A. H. Diamond, L. L. Dines, E. L. Dodd, D. M. Dribin, W. E. Edington, Churchill Eisenhart, W. H. Erskine, G. C. Evans, G. W. Evans, H. P. Evans, B. F. Finkel, C. H. Fischer, W. W. Flexner, M. M. Flood, K. W. Folley, L. R. Ford, F. J. Gerst, B. C. Getchell, H. A. Giddings, H. H. Goldstine, G. D. Gore, Cornelius Gouwens, F. L. Griffin, P. R. Halmos, Israel Halperin, D. C. Harkin, E. G. Harrell, M. C. Hartley, M. L. Hartung, J. O. Hassler, Alan Hazeltine, E. R. Hedrick, C. B. Hennel, M. R. Hestenes, R. U. Hightower, T. H. Hildebrandt, R. C. Hildner, D. L. Holl, T. R. Hollcroft, I. M. Hostetter, E. M. Hove, M. G. Humphreys, E. V. Huntington, Witold Hurewicz, W. R. Hutcherson, C. A.

Hutchinson, D. H. Hyers, M. H. Ingraham, Dunham Jackson, E. D. Jenkins, Fritz John, H. F. S. Jonah, A. J. Kempner, J. M. Kinney, S. C. Kleene, E. E. Knight, L. A. Knowler, H. L. Krall, J. H. Kusner, O. E. Lancaster, Cornelius Lanczos, G. B. Lang, R. E. Langer, C. H. Langford, C. G. Latimer, C. A. Lester, G. H. Ling, M. I. Logsdon, W. R. Longley, Brother Louis De La Salle, R. G. Lubben, E. J. McShane, C. C. MacDuffee, Saunders MacLane, H. M. MacNeille, P. T. Maker, Dorothy Manning, Anna Marm, R. H. Marquis, William Marshall, R. S. Martin, R. G. Mason, A. E. Meder, Karl Menger, H. J. Miles, A. N. Milgram, W. E. Milne, W. L. Miser, Virginia Modesitt, C. N. Moore, M. G. Moore, R. L. Moore, T. W. Moore, Marston Morse, E. J. Moulton, F. R. Moulton, G. C. Munro, S. B. Myers, A. L. Nelson, C. A. Nelson, C. J. Nesbitt, C. V. Newsom, M. J. Norris, F. C. Ogg, E. G. Olds, P. S. Olmstead, T. P. Palmer, E. W. Paxson, S. E. Pence, E. D. Pepper, P. M. Pepper, O. J. Peterson, H. P. Pettit, A. E. Pitcher, H. S. Pollard, V. C. Poor, G. B. Price, Irene Price, Tibor Radó, G. Y. Rainich, E. D. Rainville, W. C. Randels, J. F. Randolph, S. E. Rasor, F. W. Reed, Harriet Rees, Francis Regan, W. T. Reid, Haim Reingold, R. G. D. Richardson, P. R. Rider, H. L. Rietz, R. F. Rinehart, Robin Robinson, J. B. Rosenbach, J. B. Rosser, M. F. Rosskopf, E. H. Rothe, D. A. Rothrock, L. L. Runge, S. T. Sanders, R. G. Sanger, A. C. Schaeffer, O. F. G. Schilling, I. J. Schoenberg, Henry Schultz, W. E. Sewell, M. E. Shanks, W. A. Shewhart, L. S. Shively, C. G. Shover, D. T. Sigley, C. H. Smiley, M. F. Smiley, A. H. Smith, F. C. Smith, G. W. Smith, G. W. Starcher, N. E. Steenrod, H. E. Stelson, Guy Stevenson, G. R. Stibitz, E. C. Stokes, R. W. Stokes, E. B. Stouffer, Otto Szász, J. S. Taylor, M. E. Taylor, H. P. Thielman, H. S. Thurston, E. W. Titt, C. B. Tompkins, C. C. Torrance, W. J. Trijitzinsky, P. L. Trump, A. W. Tucker, Henry VanEngen, H. E. Vaughan, H. S. Wall, S. E. Warschawski, J. H. Weaver, D. L. Webb, M. J. Weiss, E. L. Welker, N. W. Wells, G. W. Whaples, J. J. Wheeler, O. L. Wheeler, G. T. Whyburn, W. M. Whyburn, D. V. Widder, L. R. Wilcox, S. S. Wilks, K. P. Williams, A. H. Wilson, Wilfrid Wilson, L. A. Wolf, M. C. Wolf, H. E. Wolfe, F. E. Wood, Kathryn Wyant, M. M. Young.

The opening meeting of the American Association for the Advancement of Science was held Monday evening in the Murat Temple. Professor E. G. Conklin delivered his retiring presidential address on the subject *Science and ethics*. Dean G. D. Birkhoff, President of the Association, served as chairman. The address was followed by a reception for all members of the Association and affiliated societies.

The meetings of the Society began Tuesday morning with two sections, Algebra and Analysis, and Topology. That afternoon, the Society met again in two sections, Analysis, and Geometry and Algebra. The presiding officers at these four sectional meetings were, respectively, Professors W. M. Whyburn, R. L. Moore, and T. H. Hildebrandt, and Dean E. B. Stouffer.

The fourteenth Josiah Willard Gibbs Lecture was delivered Tuesday afternoon at four-thirty by Charles A. Kraus, Research Professor of Chemistry at Brown University, before a joint meeting of the Society and the Association for the Advancement of Science, at which Dean G. D. Birkhoff presided. The subject was *The present status of*

the theory of electrolytes. The attendance at this lecture was about five hundred.

At the beginning of the general session Wednesday morning, President J. W. Putnam, of Butler University, gave a brief address of welcome. This general session included, also, a business meeting and the annual election. President R. L. Moore presided.

Wednesday afternoon, Professor Alonzo Church, of Princeton University, gave an address entitled *The constructive second number class* at a general session at which Professor D. R. Curtiss presided. Following this, the Society met in two sections, Mathematical Logic and Algebra. The section on Mathematical Logic was a joint session with the Association for Symbolic Logic at which Professor E. V. Huntington presided. Professor A. A. Albert presided in the Algebra section.

The Society met Thursday morning in two sections, Mathematical Statistics and Analysis. The section on Mathematical Statistics was a joint session with the Institute of Mathematical Statistics under the chairmanship of Professor W. A. Shewhart, President of the Institute. Professor E. J. McShane presided in the Analysis section.

On Thursday afternoon, at a joint session of the Society, the Mathematical Association of America, the Econometric Society, the Institute of Mathematical Statistics, and Sections A and K of the Association for the Advancement of Science, Professor G. C. Evans, of the University of California, gave an address as retiring Vice President of the Association for the Advancement of Science and Chairman of Section A. The subject was *Mathematical progress in theoretical economics*. Professor W. D. Cairns, Vice President of the Association for the Advancement of Science and Chairman of Section A, presided.

Sessions of the Mathematical Association of America were held on Thursday afternoon and Friday morning.

A joint dinner for members of the mathematical organizations and their guests was held on Wednesday evening in the Marott Hotel. The attendance was three hundred twenty-five. Dean G. D. Birkhoff, Dr. F. R. Moulton, Dr. J. McKeen Cattell, and Miss Martha Hildebrandt were guests of honor. Professor R. E. Langer was toastmaster. In recognition of the close ties of the mathematical organizations with the American Association for the Advancement of Science, he introduced, as the first speaker, Dean G. D. Birkhoff, President of that Association, followed by the Permanent Secretary, Dr. F. R. Moulton. Two speakers, Professors G. C. Evans and T. H. Hildebrandt, represented the mathematicians. Professor G. T. Whyburn

presented resolutions, which were adopted unanimously, thanking the President and members of the Mathematics Department of Butler University, the local committee, and the Marott Hotel for their excellent arrangements and cordial hospitality.

On Thursday afternoon, following the Association meeting, a tea was given in the Recreation Room, Jordan Hall. The hosts and hostesses were President and Mrs. J. W. Putnam, Dean and Mrs. G. A. Ratti of Butler University, and the members of the Butler University Department of Mathematics.

The meeting of the Board of Trustees at noon, December 28, in Room 127, Jordan Hall, did not have a quorum present. An adjourned meeting, therefore, was held on January 2 at 7:30 P.M. in the Office of the Society in New York City.

The Council held a meeting in the Marott Hotel on Tuesday evening, December 28, following dinner. There was also an adjourned meeting of the Council at 1 P.M., Thursday, December 30, in Jordan Hall.

At the business meeting, Wednesday morning, December 29, the annual election closed. Announcements were made of the results of the election and of the decisions of the Council in various matters.

Professor William Marshall and Dr. E. R. Lorch were tellers for the election. A total of two hundred fifty-four ballots were cast. The following officers and members of the Council were declared duly elected:

Vice Presidents, Professors R. E. Langer and J. F. Ritt.

Secretary, Dean R. G. D. Richardson.

Associate Secretaries, Professors W. L. Ayres and M. H. Ingraham, and Dean T. M. Putnam.

Treasurer, Professor B. P. Gill.

Members of the Editorial Committee of the Bulletin, Professors L. M. Graves and P. A. Smith.

Member of the Editorial Committee of the Transactions, Professor Einar Hille.

Member of the Editorial Committee of the Colloquium Publications, Professor J. R. Kline.

Member of the Editorial Board of the American Journal, Professor T. H. Hildebrandt.

Members of the Council, Professors Philip Franklin, A. D. Michal, John von Neumann, J. L. Synge, and W. J. Trjitzinsky.

It was announced that the following fifty-two persons had been elected to membership in the Society:

Professor Emil Artin, University of Notre Dame;

Mr. Frank Boehm, Manager, Life Insurance, New York, N. Y.;
Professor Caldwell Elwood Boulware, Barber-Scotia College, Concord, N. C.;
Mr. Reynolds Driver Brown, Philadelphia Storage Battery Company, Philadelphia, Pa.;
Mr. Robert Buchanan, Golden Gate College, San Francisco, Calif.;
Mr. Everett Tyler Burton, Bell Telephone Laboratories, New York, N. Y.;
Miss Mary Agnes Campbell, Lamar College, Beaumont, Texas;
Professor Clifton O'Neil Carey, University of Michigan;
Mr. Charles Lemuel Carroll, University of North Carolina;
Miss Esther A. Compton, Wood Junior College, Mathiston, Miss.;
Professor John Cleveland Cothran, Duluth State Teachers College, Duluth, Minn.;
Dr. Ainsley H. Diamond, University of California at Los Angeles;
Dr. Churchill Eisenhart, University of Wisconsin;
Professor Hilbert Adam Fisher, North Carolina State College, Raleigh;
Mr. George Nelvin Garrison, College of the City of New York;
Mr. Paul Richard Halmos, University of Illinois;
Mr. George Madison Harner, Marquette University, Milwaukee, Wis.;
Mr. Richard Manning Hermes, University of Santa Clara, Santa Clara, Calif.;
Dr. Doncaster George Humm, Los Angeles, Calif.;
Miss May Baggott Kelly, Bulkeley High School, Hartford, Conn.;
Mr. James Ronald Fraser Kent, University of Illinois;
Dr. Thomas J. Killian, Barkon-Tube Lighting Corporation, Long Island City, N. Y.;
Mr. Charles Henry Lehmann, Cooper Union, New York, N. Y.;
Rev. James Luke McKenney, O.P., Providence College, Providence, R. I.;
Professor Michel George Malti, Cornell University;
Mr. Ralph Mansfield, Armour Institute of Technology;
Miss Lillian Moore, Far Rockaway High School, Far Rockaway, N. Y.;
Mr. Marvin G. Moore, Indiana University;
Dr. David Carruthers Murdoch, Yale University;
Professor James O. Patterson, St. Charles Seminary, Philadelphia, Pa.;
Professor Robert Leonidas Peguese, Allen University, Columbia, S. C.;
Professor George Whitefield Petrie, South Dakota State School of Mines, Rapid City;
Dr. Harriet Rees, University of Utah;
Dr. Robert Weir Rempfer, Rensselaer Polytechnic Institute;
Mr. William Garfield Riley, Washington, D. C.;
Dr. Erich H. Rothe, Penn College, Oskaloosa, Iowa;
Professor Phillip Justin Rulon, Harvard University;
Mr. George Sandalls, Bell Telephone Laboratories, New York, N. Y.;
Dr. Arthur Sard, Queens College, Flushing, N. Y.;
Dr. James K. Senior, University of Chicago;
Mr. Samuel Schwartz Smith, University of Utah;
Dr. David Barnard Steinman, New York, N.Y.;
Mr. Charles Edward Stevens, Hofstra College, Hempstead, N. Y.;
Dr. George Robert Stibitz, Bell Telephone Laboratories, New York. N. Y.;
Dr. George Booth Van Schaack, University of Rochester;
Mr. Holmes Hinckley Welch, Brooks School, North Andover, Mass.;
Dean Daniel E. Whelan, Loyola University of Los Angeles;
Mr. Harold C. Wiedeman, University of Denver;
Mr. Charles Wiley Williams, Washington and Lee University;

Dean Claude Rodolphus Wood, State Teachers College, Jacksonville, Ala.;
Mr. Richmond T. Zoch, Weather Bureau Office, Washington, D. C.;
Mr. Max Zwillinger, U. S. Navy Yard, Washington, D. C.

It was reported that the following had been elected as nominees on the Institutional Memberships of the various institutions and departments:

Brown University: Messrs. Edward Maurice Beesley and Mahlon Marsh Day, Dr. Douglas Derry, and Mr. Charles Samuel Sutton.

University of California at Los Angeles: Messrs. Robert Henry Sorgenfrey and Paul Alexander White.

Department of Mathematics, University of Chicago: Mr. Leonidas Alaoglu, Miss Evelyn R. Garbe, and Messrs. Sam Perlis and Leonard Tornheim.

Members of the Department of Mathematics of the College of the City of New York: Mr. Julian H. Blau.

Columbia University: Messrs. Ellis Robert Kolchin and Howard Levi, Miss Ethel Harriet Raybould, Miss Leona Rosenthal, and Messrs. Rubin Schatten and Walter Moakler Wynne.

Cornell University: Messrs. Charles Erwin Clark, Edwin Alfred Sydney Galbraith, Walter Joel Harrington, Karl Johannes, Donald S. Miller, Milton Everett Terry, George Brinton Thomas, and William Dean Wray.

Duke University: Messrs. Grady Craven Frank and William Gillespie McGavock.

Department of Mathematics, University of Illinois: Messrs. Warren Ambrose, Ross Allen Beaumont, James Dow Campbell, C. Ronald Cassity, Foster Leroy Dennes, James McQueen Dobbie, and Joseph J. Eachus, Miss Margaret Mary Hansman, Messrs. Donald Eugene Kibbey, Andrew O. Lindstrum, Philip Tillotson Maker, Carl E. Marshall, Donald Rhea Shreve, David Wright Starr, Everett Linus Welker, and F. Joachim Weyl.

Institute for Advanced Study: Professors Erich Hecke (Mathematisches Seminar der Universität, Hamburg), Václav Hlavatý (University of Prague), and Tadasi Nakayama (Osaka Imperial University), Dr. Cecil James Nesbitt, Mr. Maxwell Herman Alexander Newman, and Dr. Frank Smithies.

Department of Mathematics, State University of Iowa: Messrs. Wayne Wallace Gutzman, Richard Albert Miller, Henry Vernon Price, Fred D. Rigby, and Carl Elof Sealander.

University of Kentucky: Mr. William Hicks Pell.

Northwestern University: Messrs. Clifford Gunnar Erickson, Howard K. Hilton, and Harold E. Wilson.

University of Pennsylvania: Mr. William Nathan Huff.

College of Saint Thomas: Mr. Michael Joseph Norris.

Stanford University: Messrs. Charles R. Bubb, Harry Earl Goheen, and Charles Douglas Olds.

University of Virginia: Mr. Alexander Doniphan Wallace.

University of Washington: Messrs. William Dwight Duthie and Richard Edward Johnson.

Department of Mathematics, Wayne University: Mr. I. Theodore Weinberg.

Department of Mathematics, Wellesley College: Martha Hathaway Plass.

Wesleyan University: Miss Mary Jane Kingman.

Western and Southern Life Insurance Company: Mr. Clifford Hartman Folz.

University of Wisconsin: Messrs. Randolph Hudie Cole, Cornelius Joseph Everett,

Buford Echols Gatewood, Wilbur Charles McDaniel, and Wilbur Leonard Mitchell.

Department of Mathematics, Yale University: Messrs. Robert Abraham Rosenbaum, Bryce Townsend Ruley, Irving Ezra Segal, and Eric Avern Sturley.

The Secretary reported that Mr. Rama Shankar Varma, M.Sc., of Christ Church College, Cawnpore, India, and Mr. John Conrad Jaeger, M.A., Lecturer, University of Tasmania, Hobart, Tasmania, have been admitted to membership in the Society in accordance with the reciprocity agreement with the London Mathematical Society.

The Secretary reported that the membership in the Society is now 2,127, including 127 contributing members, 175 nominees of institutional members, and 78 life members. There are also 88 institutional members. The total attendance of members at all meetings in 1937 was 1337; the number of papers read was 430; the number of hour lectures was 16; the number of members attending at least one meeting was 784.

The reports of the Treasurer and of the Auditors (Mr. S. A. Joffe and Professor A. E. Meder) showed a balance of $18,142.71 for the year ending November 30, 1937, exclusive of the balances in the Colloquium Fund, Sinking Fund, and special funds. From this balance several printing bills for 1937 must be paid. It was pointed out that the expenses for 1937 are considerably below the total figures for 1935 and 1936 because, during 1935 and 1936, the Society acted as agent for the expenditure of money for extra activities not normally included in the budget. The regular budgetary expenditures have increased only slightly during the last few years. The Society's investments including Endowment Fund, Sinking Fund, and other special funds had a market value on November 30, 1937, of $90,396.75. The net interest income for the year was $3,796.64; dues from institutional memberships amounted to $6,429.75; dues from contributing memberships, $1,201.97; and dues from ordinary memberships were $12,873.54. The Colloquium Fund showed a balance of $9,806.32. The total received from the sale of the Society's publications was $10,029.00. The expenditures included a grant of $2,500.00 to the American Journal of Mathematics which is published jointly by the Society and the Johns Hopkins University. The trustees adopted a budget for 1938 showing estimated expenditures and receipts as $37,150 and $38,346.07, respectively. These items do not include a special budget for the Semicentennial Celebration.

The Librarian reported that the Library of the Society now contains 9,069 volumes.

The following appointments were reported: as auditors of the

accounts for 1937, Mr. S. A. Joffe and Professor A. E. Meder, as representative on the National Research Council for the period beginning July 1, 1938, and to succeed Professor J. H. Van Vleck, Professor J. F. Ritt; as representatives on the Council of the American Association for the Advancement of Science for the year 1938, Professors R. E. Langer and F. D. Murnaghan; as committee to study the question of publicity for mathematics, Professor C. N. Moore (chairman), Dr. Saunders MacLane, and Professor G. B. Price.

The Bôcher Prize is to be awarded at the 1938 Annual Meeting to the author of a paper on Analysis appearing during the period 1933–1937.

A report from the Organizing Committee of the International Congress of Mathematicians states that the following subcommittees have been appointed: Budget Committee, Professors M. H. Ingraham (chairman), W. C. Graustein, Einar Hille, Marston Morse, Dean R. G. D. Richardson, and Professor D. V. Widder; Committee on Cooperation, Professors M. H. Stone (chairman), G. A. Bliss, Dr. E. R. Hedrick, Professors Dunham Jackson and Oswald Veblen; Entertainment Committee, Professors D. V. Widder (chairman), C. R. Adams, L. P. Copeland, and Philip Franklin, Dr. W. T. Martin, Dean R. G. D. Richardson, Mesdames G. D. Birkhoff, W. C. Graustein, and H. B. Phillips; Publicity Committee, Professors Philip Franklin (chairman), R. C. Archibald, and E. T. Bell, Mr. Garrett Birkhoff, and Professor Arnold Dresden. Plans are going forward satisfactorily. A budget for the early part of 1938 was adopted. The following sections have been tentatively authorized: (I) Algebra and Theory of Numbers; (II) Analysis; (III) Geometry and Topology; (IV) Probability, Statistics, Actuarial Science, Economics; (V) Mathematical Physics, Mechanics, Astronomy, Geophysics; (VI) Logic, Philosophy, History, Didactics.

The Rockefeller Foundation, the Institute for Advanced Study, and the National Research Council were thanked for generous subscriptions in support of the International Congress of Mathematicians.

The report presented by Professor Ingraham as chairman of the Committee on Fees at Meetings was adopted. This report recommended that Colloquium fees be abolished and that, in their place, registration fees for summer meetings be substituted. The registration fee would ordinarily be one dollar for each member of the organizations and fifty cents for each additional member of his family and for each guest.

The following resolution regarding the death of Professor Morley was adopted:

The Council of the American Mathematical Society wishes to record its deep sense of loss at the death of Professor Frank Morley of the Johns Hopkins University. Those who knew him will long recall his gentlemanly dignity and unfailing courtesy. Professor Morley was one of the first members of the Society and lived to see it grow from modest beginnings to its present position as one of the leading mathematical organizations of the world. He served as editor of its Bulletin for several years and as its president during the years 1919–20. His services as editor-in-chief of the American Journal of Mathematics covered a long period of years, during part of which time the Society cooperated with the Johns Hopkins University in its publication. An account of his services to the Society and of his scientific abilities and attainments will appear in an early number of the Bulletin.

Additional times and places of meetings in 1938 were fixed as follows: Berkeley, April 9; New York, October 29; Cleveland, November 25–26; Los Angeles, November 26. Invitations were received from Ohio State University and Louisiana State University for the Annual Meeting in 1939.

It was announced that the October, 1938, number of the American Journal of Mathematics is to be dedicated to G. W. Hill, a former president of the Society, the one hundredth anniversary of whose birth takes place during 1938. Papers from leading mathematicians in fields allied to that of Dr. Hill will appear in this number.

Titles and cross references to the abstracts of papers read at the regular sessions follow below. The papers were read as follows: papers numbered 1 to 8 in the section for Algebra and Analysis on Tuesday morning; papers 9 to 16 in the section for Topology on Tuesday morning; papers 17 to 22 in the section for Analysis on Tuesday afternoon; papers 23 to 27 in the section for Geometry and Algebra on Tuesday afternoon; papers 28 to 33 in the general session on Wednesday morning; papers 34 to 39 in the section for Mathematical Logic on Wednesday afternoon; papers 40 to 45 in the section for Algebra on Wednesday afternoon; papers 46 to 51 in the section for Mathematical Statistics on Thursday morning; papers 52 to 59 in the section for Analysis on Thursday morning; and papers 60 to 93, whose abstract numbers are followed by the letter *t*, were read by title. Of those presenting papers, Dr. Olaf Helmer was introduced by Professor Rudolf Carnap, Miss Audrey Wishard by Professor L. V. Ahlfors, and Professor Eugene Stephens by Professor Francis Regan. Paper 4 was presented by Professor G. A. Bliss, paper 9 by Dr. M. F. Smiley, paper 23 by Dr. S. B. Myers, and paper 47 by Professor H. T. Davis.

1. *A relative of the lemma of Schwarz*, by E. F. Beckenbach. (Abstract 43-11-392.)

2. *The completion of a Boolean ring and its application to integration*, by H. M. MacNeille. (Abstract 44-1-36.)

3. *Some existence theorems in the calculus of variations. I. Free problems*, by E. J. McShane. (Abstract 44-1-86.)

4. *Fields for multiple integrals of the calculus of variations*, by G. A. Bliss, M. F. Smiley, and Leonidas Alaoglu. (Abstract 44-1-8.)

5. *A general existence theorem for simultaneous equations*, by A. H. Copeland. (Abstract 44-1-13.)

6. *The remainder term for approximations of linear type*, by W. E. Milne. (Abstract 44-1-40.)

7. *Algebraic theory of abelian functions*, by O. F. G. Schilling. (Abstract 44-1-57.)

8. *An elementary derivation of the Uspensky quadrature formulas*, by C. C. Torrance. (Abstract 44-1-88.)

9. *Metric lattices*, by M. F. Smiley and L. R. Wilcox. (Abstract 44-1-63.)

10. *An existence theorem and some applications to topology*, by A. N. Milgram. (Abstract 44-1-38.)

11. *Symmetric and alternating products of circles*, by A. W. Tucker. (Abstract 44-1-71.)

12. *Complexes whose vertices have manifolds as linked complexes* (preliminary report), by W. W. Flexner. (Abstract 44-1-19.)

13. *Concerning upper semicontinuous collections and the decomposition of points of normal spaces*, by R. G. Lubben. (Abstract 44-1-32.)

14. *Theory of the topological order in linear topological spaces*, by E. H. Rothe. (Abstract 44-1-56.)

15. *On the characterization of abstract spaces by postulating the existence or non-existence of certain types of metrics*, by H. E. Vaughan. (Abstract 44-1-72.)

16. *On the postulates for linear topological spaces*, by E. W. Paxson. (Abstract 44-1-47.)

17. *The Vitali covering theorem for Carathéodory linear measure*, by J. F. Randolph. (Abstract 43-11-386.)

18. *On expansions in solutions of differential equations*, by M. G. Moore. (Abstract 44-1-42.)

19. *Theory of functions of a complex variable defined over general sets*, by W. J. Trjitzinsky. (Abstract 44-1-70.)

20. *On the one dimensional Green's function*, by W. M. Whyburn. (Abstract 44-1-78.)

21. *The theory of integration*, by G. B. Price. (Abstract 44-1-48.)

22. *On secant and tangent coefficients*, by D. C. Harkin. (Abstract 44-1-22.)

23. *The group of isometries of a Finsler manifold*, by S. B. Myers and N. E. Steenrod. (Abstract 44-1-44.)

24. *Deformations of the inner equator of a torus*, by C. B. Tompkins (National Research Fellow). (Abstract 44-1-69.)

25. *Abelian hypergroups* (preliminary report), by H. S. Wall. (Abstract 44-1-73.)

26. *The maximum number of contacts of two algebraic surfaces*, by T. R. Hollcroft. (Abstract 44-1-25.)

27. *Matric conjugates in a ring $R(A)$*, by H. S. Thurston. (Abstract 44-1-68.)

28. *Interior transformations on 2-dimensional manifolds*, by G. T. Whyburn. (Abstract 44-1-77.)

29. *Some inequalities in the theory of Fourier series*, by Otto Szász. (Abstract 44-1-67.)

30. *Quadratic null forms over a function field*, by A. A. Albert. (Abstract 44-1-6.)

31. *Metric spaces and positive definite functions*, by I. J. Schoenberg. (Abstract 44-1-60.)

32. *A theorem on relations and its applications to covering theorems of topology*, by Karl Menger. (Abstract 44-1-37.)

33. *Dirac equations and conditional invariants*, by G. Y. Rainich. (Abstract 44-1-49.)

34. *On the consistency of Quine's "New foundations for mathematical logic,"* by J. B. Rosser. (Abstract 44-1-54.)

35. *The syntax of a language with infinite expressions*, by Olaf Helmer. (Abstract 44-1-23.)

36. *A theorem on deducibility for second-order functions*, by C. H. Langford. (Abstract 44-1-30.)

37. *A set of postulates for an algebra of n-valued logic*, by D. L. Webb. (Abstract 44-1-75.)

38. *On the categoricity of the postulates for Boolean algebra*, by Paul Henle. (Abstract 44-1-24.)

39. *On the reduction of Gentzen's calculus LJ*, by H. B. Curry. (Abstract 44-1-14.)

40. *The transformation of bases of relative linear sets*, by Margarete C. Wolf. (Abstract 44-1-80.)

41. *Modularity in the theory of lattices*, by L. R. Wilcox. (Abstract 44-1-79.)

42. *Strict equivalence of matric pencils* (preliminary report), by M. M. Flood. (Abstract 44-1-20.)

43. *Note on almost-universal forms*, by P. R. Halmos. (Abstract 43-11-390.)

44. *Commutative algebras which are polynomial algebras*, by R. F. Rinehart. (Abstract 44-1-52.)

45. *The uniqueness of the power series representation of certain fields and valuations*, by Saunders MacLane. (Abstract 44-1-35.)

46. *On a fiducial test of the significance of the difference of the means of two normally distributed populations which are not known to have equal variances*, by Daisy M. Starkey. (Abstract 44-1-65.)

47. *Some results in the theory of random series*, by H. T. Davis and H. E. Jones. (Abstract 44-1-15.)

48. *A moment-generating function which is useful in solving certain matching problems*, by E. G. Olds. (Abstract 43-11-428.)

49. *A rating table for card-matching experiments*, by E. V. Huntington. (Abstract 44-1-84.)

50. *Note on the probability distribution of a product*, by B. H. Camp. (Abstract 44-1-10.)

51. *The power function of the χ^2-test*, by Churchill Eisenhart. (Abstract 44-1-18.)

52. *On the degree of approximation in some convergence theorems in conformal mapping*, by S. E. Warschawski. (Abstract 44-1-74.)

53. *The analog to the Cauchy integral and circulation functions of the second kind*, by V. C. Poor. (Abstract 43-11-430.)

54. *The interrelations among the fundamental solutions of the generalized hypergeometric equation when $p=q+1$. I. Non-logarithmic cases*, by F. C. Smith. (Abstract 43-11-432.)

55. *A theorem on generalized Jacobians*, by Tibor Radó. (Abstract 43-11-388.)

56. *Jackson summation of the Faber development*, by W. E. Sewell. (Abstract 44-1-61.)

57. *Some results concerning the behavior at infinity of real continuous solutions of algebraic difference equations*, by O. E. Lancaster. (Abstract 43-11-427.)

58. *On the absolute summability of Fourier series*, by W. C. Randels. (Abstract 44-1-50.)

59. *Some conditions on functions of bounded type*, by Audrey Wishard. (Abstract 44-1-89.)

60. *Polynomial approximations for elliptic functions*, by E. T. Bell. (Abstract 44-1-7-*t*.)

61. *Second order partial differential operators of elliptic type. Closure*, by J. W. Calkin. (Abstract 44-1-9-*t*.)

62. *Concerning Laplace transforms of periodic functions*, by R. V. Churchill. (Abstract 44-1-11-*t*.)

63. *A line transformation related to reciprocal polars of a quadric surface*, by J. M. Clarkson. (Abstract 44-1-12-*t*.)

64. *The inverse of Meusnier's theorem in the geometry of element series*, by J. J. De Cicco. (Abstract 44-1-16-*t*.)

65. *Contractible problems of Bolza*. II, by W. L. Duren. (Abstract 44-1-17-*t*.)

66. *Extension formulas for interpolation with leading differences*, by C. H. Forsyth. (Abstract 43-11-391-*t*.)

67. *On hypergroups, multigroups, and product systems*, by Lois W. Griffiths. (Abstract 44-1-21-*t*.)

68. *The transformation of statistics to simplify their distributions*, by Harold Hotelling and L. R. Frankel. (Abstract 44-1-26-*t*.)

69. *The classification of element transformations by means of isogonal and equi-tangential series*, by Edward Kasner and J. J. De Cicco. (Abstract 44-1-27-*t*.)

70. *The geometry of the whirl-motion group G_6. Elementary invariants*, by Edward Kasner and J. J. De Cicco. (Abstract 44-1-28-*t*.)

71. *Geometry in an n-dimensional space with the arc length* $s=\int(A_ix''^i+B)^{1/p}dt$, by Akitsugu Kawaguchi. (Abstract 43-11-389-*t*.)

72. *Non-linear algebraic difference equations with formal solutions of the same type as the formal solutions of linear homogeneous difference equations*, by O. E. Lancaster. (Abstract 44-1-29-*t*.)

73. *On the equivalence of the nilpotent elements of a semi-simple ring*, by Jakob Levitzki. (Abstract 44-1-31-*t*.)

74. *Subrings of direct sums and related topics*, by N. H. McCoy and Deane Montgomery. (Abstract 44-1-33-*t*.)

75. *On the simultaneous approximation of a function and its derivatives by sums of Birkhoff type*, by W. H. McEwen. (Abstract 44-1-34-*t*.)

76. *On linear subsets of metric spaces*, by Karl Menger and A. N. Milgram. (Abstract 44-1-39-*t*.)

77. *On expansions in series of exponential functions*, by M. G. Moore. (Abstract 44-1-41-*t*.)

78. *A continuous function with no unilateral derivatives*, by A. P. Morse. (Abstract 44-1-43-*t*.)

79. *Relations between the coefficients of the modular representations of groups*, by C. J. Nesbitt. (Abstract 44-1-45-*t*.)

80. *Non-singular linear combinations of general forms*, by Rufus Oldenburger. (Abstract 44-1-46-*t*.)

81. *The Cauchy singular integral and circulation functions*, by V. C. Poor. (Abstract 43-11-429-*t*.)

82. *A theorem on quadratic forms*, by W. T. Reid. (Abstract 44-1-51-*t*.)

83. *On periodic continued fractions*, by W. C. Risselman. (Abstract 44-1-53-*t*.)

84. *On the transformation group for diabolic magic squares of order four*, by J. B. Rosser and R. J. Walker. (Abstract 44-1-55-*t*.)

85. *A note on infinite perfect fields*, by O. F. G. Schilling. (Abstract 44-1-58-*t*.)

86. *On the structure of local class field theory*, by O. F. G. Schilling. (Abstract 44-1-59-*t*.)

87. *Remarks on the foundations of geometry*, by A. R. Schweitzer. (Abstract 43-11-387-*t*.)

88. *Note on the Faber coefficients of a continuous function*, by W. E. Sewell. (Abstract 43-11-431-*t*.)

89. *Affine differential geometry of plane curves from the point of view of Wilczynski*, by H. A. Simmons. (Abstract 44-1-62-*t*.)

90. *Torsion of regions bounded by circular arcs*, by I. S. Sokolnikoff and Elizabeth S. Sokolnikoff. (Abstract 44-1-64-*t*.)

91. *The purely operational solution of linear differential equations with polynomial coefficients*, by Eugene Stephens. (Abstract 44-1-66-*t*.)

92. *Analytical methods in hypercomplex systems*, by Olga Taussky. (Abstract 43-11-433-*t*.)

93. *Interior transformations on certain curves*, by G. T. Whyburn. (Abstract 44-1-76-*t*.)

T. R. HOLLCROFT,
Associate Secretary

FRANK MORLEY—IN MEMORIAM

With the death of Professor Frank Morley in Baltimore on October 17, 1937, there passed one of the more striking figures of the relatively small group of men who initiated that development which, within his own lifetime, brought mathematics in America from a minor position to its present place in the sun. His contribution to this development, through his own untiring research, his scholarly treatises, his wide guidance, and his inspiring teaching, has been most notable.

Frank Morley was born at Woodbridge, Suffolk, England on September 9, 1860, the son of Joseph R. and Elizabeth (Muskett) Morley. He took his A.B. degree at King's College, Cambridge, in 1884, his A.M. degree in 1887. During the period 1884–1887 he served as master in Bath College, England.

His mathematical career in America, extended over a period of fifty years, began with an appointment to an instructorship at Haverford College in 1887. The following year he was promoted to a professorship, a position which he held for twelve years. This was perhaps his most productive period, being marked by the appearance (in collaboration with James Harkness at Bryn Mawr) of two volumes on the theory of functions, and of about twenty of his fifty articles. In 1898 he received the degree of Doctor of Science from Cambridge University.

The period at Haverford was also most eventful in his family life. On July 11, 1889, he married Lilian Janet Bird of Hayward's Heath, Sussex, England. Their three children, Christopher D., Felix M., and Frank V., all were born at Haverford in the decade following.*

The graduate work in mathematics at the Johns Hopkins University, so brilliantly inaugurated by Sylvester, had just before 1900 sunk into a decline, partly because of the poor health of Professor Craig. The death of Craig in the spring of that year, and the retirement of Simon Newcomb from a sort of absentee headship, forced President Gilman to look elsewhere for leadership in this work. With his usual happy judgment of men and their capacities, he invited Professor Morley to become professor of mathematics and head of the department. This carried with it the editorship of the American Journal of Mathematics, at that time under the sole control of the University. That he was able to revive the department, to attract, retain, and train graduate students in adequate numbers, and to maintain over a long period of lean years an active mathematical center, is perhaps the best evidence of his executive ability and his inspiring leadership. This was accomplished in the face of competition from many departments much more amply supported and manned, even though for many years the only form of graduate student aid at his disposal was a single fellowship.

He has said that he entered on his career at Johns Hopkins with some misgivings. It entailed, of course, great changes in his family and social life. The entire content of his teaching had to be raised to the graduate level. On the scientific side he had to follow, in great measure at least, not the paths which he might have chosen for him-

* It may be that some of the Bulletin readers do not know of this unusual family. All three of the brothers held Rhodes scholarships at New College, Oxford, a family record which is likely to stand. Also, all three have attained distinction in literature. Christopher Morley is the well known novelist and critic; Felix Morley, now editor of the Washington Post, is an authority on current political and international problems; and Frank V. Morley, a Ph.D. in mathematics of Oxford University (1923), is an author and publisher with headquarters in London.

self, but rather those along which his students could or should follow him. But apparently the transition was not difficult. He and Mrs. Morley settled easily into the life of the University and the city. When I returned to the University in 1904 as his first instructor, he had acquired an honorable place in that company of individualists which made up the Johns Hopkins faculty, and also a comfortable chair for his regular game of bridge at the down town University Club of Baltimore.

For a number of years Professor Morley participated in research programs of the Carnegie Institution, assisted by various associates including H. Bateman, J. R. Conner, and myself. When he retired from active university service in 1928, he continued his work as research assistant of this Institution. In 1933 he published, in collaboration with his son, Frank V. Morley, a notable volume on *Inversive Geometry*. His last article (with J. R. Musselman) appeared in the American Journal of Mathematics, October, 1937, the month of his death.

Professor Morley was a member of the New York Mathematical Society, and of its successor, the American Mathematical Society, from about the time of organization. He was editor of this Bulletin during the period 1895–1898, and from October 1899 through 1902. He was vice president of this Society in 1902, and president in the 1919–1920 biennium. He served as cooperating editor of the American Journal of Mathematics in 1899–1900 and in 1929–1937, and as editor in the twenty-one year interval, 1900–1921. In 1921 he sponsored the movement for joint control of the Journal by the John Hopkins University and this Society, and served as a member of the joint editorial board over the period 1921–1928. This type of editorial coöperation has since been adopted by other mathematical journals. He was also a member of the London Mathematical Society, the Circolo Matematico di Palermo, the American Philosophical Society, and the American Academy of Arts and Sciences.

The major contributions of Professor Morley to mathematical science are the treatises of which he was a joint author. The first of these, *A Treatise on the Theory of Functions* (Macmillan, 1893), was a most ambitious undertaking. It aimed to give to American and English readers an account of that great body of doctrine which had been developed in the main by continental mathematicians. That the need for such a book was widely felt is shown by the appearance, somewhat earlier in the same year, of Forsyth's *Theory of Functions of a Complex Variable* (Cambridge University Press). For a number of years these two books served English-speaking students as standard introductions to this field. Five years later Harkness and Morley published their *Introduction to the Theory of Analytic Functions* (Macmillan, 1898). This admirably written book still maintains its place as a classic.

Professor Morley was deeply versed in the metric and projective geometry, the invariant algebra, and the kinematical and physical theory which constituted the major portion of the scholastic mathematics of his time. A fairly large proportion of his papers deal with the geometry of kinematic problems. It was quite natural that his early work in function theory should suggest an exploration of the geometry in the complex plane. This led him, not merely to many new theorems, but also to elegant expositions, within a wide field of metric geometry. It constitutes perhaps his greatest individual achievement. His work in this direction, together with a great deal of related material, is collected in the stimulating volume, *Inversive Geometry* (with Frank V. Morley, 1933). One quotation from the authors' preface is illuminating:—"We believe that the tradition that simple geometrical and mechanical questions are to be handled only as Euclid or Descartes might have handled them is very hampering; that the ideas of Riemann, Poincaré, Klein, and others have pleasant reverberations in the investigation of elementary questions by students of proper maturity and leisure."

His papers on projective geometry are models of precise and resourceful thought

and of concise and artistic exposition. They deal with rather clearly defined situations and objectives. He used to say that every author owed to the reader at least one good illustrative example. He usually conceded to the reader enough imagination to take care of the nth case if given a good start. Thus in *Projective coordinates*, Transactions of this Society, vol. 4 (1903), §10, referring to a planar theorem of Dr. Hun, he says:—"I wish to prove this fact in such form that the restriction to two dimensions falls away." Again in the paper *On the extension of a theorem of W. Stahl*, Cambridge Congress, 1912, the extension in question reads as follows (ρ_2^m being a rational planar m-ic): "For the curve ρ_2^m the curve of lowest class on the double lines is of class $2(m-3)$; the common lines of this and ρ give an $I_{m-3}^{4(m-3)}$; and my point is that this is *the dual form of the counter-curve*." He goes on to say: "I shall give the proof in the case of ρ_2^5 by a method which applies to all cases." The case $m=5$ here chosen is the first really illustrative case, since Stahl's $m=4$ is too simple even to indicate the extension. Professor White thinks most highly of the paper *On the geometry whose element is the 3-point of a plane*, Transactions of this Society, vol. 4 (1903), but my own favorite is the four-page article *On the Lüroth quartic curve*, American Journal of Mathematics, vol. 41 (1919). It is a penetrating geometric analysis of kaleidoscopic character which eventually yields an algebraic result quite unattainable by conventional methods. Two articles in the Mathematische Annalen, vols. 49, 51, on ruler constructions for a linear covariant of the quintic and for certain polars should not be overlooked. One remarkable theorem might be singled out (Proceedings of the London Mathematical Society, (2), vol. 2 (1904)): A quintic curve on the nine flexes of a cubic curve and on the twelve other intersections of the lines of flexes has forty-five flex tangents which pass by nines through five points on the curve.

Professor Morley's chief contribution to algebra is in the field of elimination (American Journal of Mathematics, vols. 47, 49). The first paragraph of this first paper is worth quoting as an example of his happy way of relating the past to the future (references are omitted):

"Salmon, considering a plane curve and its tangent at a point x, wrote a connex which cuts out the remaining intersections of the tangent and the curve. It is of the form $A^2x^{2(m-1)}y^{m-1}$ when the curve is Ax^{m+1}. A^3 indicates that the coefficients are of the third degree in the coefficients A.

"The connex naturally occurs in the theory of Abelian integrals of the third kind.

"If now we remark that when the curve has a double point x, y becomes arbitrary, we have at once from Salmon's connex $m(m+1)/2$ curves of order $2m-2$ on the double point.

"As all first polars are also on the double point, we have 3 curves of order m which give $3(m-1)m/2$ independent curves of order $2m-2$, so that in all we have a basis of $m(2m-1)$ curves of order $2m-2$, whence the discriminant is written down as a determinant. What is necessary then is to adapt Salmon's connex to a net which is not a net of first polars."

This he proceeds to do and thus writes the ternary eliminant for three curves of the same order as a determinant. In the second paper (written jointly with myself) the syzygy used is extended to all orders and any number of variables. Its range of applicability is however limited so that it effects only two-thirds (roughly) of all cases of ternary elimination, with a reduced range as the dimension increases.

I think our members would enjoy reading, and even rereading, the philosophic observations and practical hints in his retiring address entitled *Pleasant questions and wonderful effects* (this Bulletin, vol. 27 (1921), pp. 309–312). In the same vein is the following rough draft of some of his remarks at a dinner given in his honor at the time of his retirement.

"The people of this country are alert and are usually intelligent. They are capable

without any strain of a far wider knowledge of mathematics than they actually possess. As a game or as a form of poetry the science should and could appeal not only to a bunch of specialists but to any one with a mind at once active and deliberate. All that is necessary is to let the fascination appear in the early stages of teaching. To this end some of the brotherhood of teachers must be cranks and not cogs.

"It is becoming essential that the delights of controlled imagination should be broadcast. For this seems the readiest way to sugar the coming of the new feudalism, the division of the universe into big-business; and to avoid a quite probable danger, that universities become one division of big business."

Professor Morley was a great teacher. It was his custom to lecture four or five times a week, and for many years all the students in the department, whether first or fourth year men, attended these lectures. In this situation his unusual talent for the application of advanced ideas to more elementary topics, and for striking translation of the abstract into the concrete, enabled him to interest and stimulate the entire group. In those days the standard test of progress was the fractional part of the lecture which was understood. As a rule the students met his challenge and ultimately attained a fair approximation to unity. He had a way of sensing any undue tension in his audience and of relieving it by a charmingly humorous remark.

Of the many who attended these lectures, forty-five took the doctorate under his immediate direction. It was a cardinal point with him to have on hand a sufficient variety of thesis problems to accommodate particular tastes and capacities. Many promising ideas which occurred to him were laid aside for student use. He followed the development of each one with great solicitude and felt fully rewarded when some evidence of independent thinking appeared. Each student was to him also a personality which he felt impelled to understand. The pleasant hospitality extended by Mrs. Morley to his young people, which will always be a delightful memory to the many who enjoyed it, helped him greatly in this endeavor. As a whole the departmental life presented an interesting adaptation of the English university system to the American scene.

Professor Morley had a strong conviction that geometry was the ideal medium for the presentation of abstract mathematical ideas to the general intellectual public and he welcomed the opportunities which came to him to spread this gospel in other universities.

Though mathematics was the chief preoccupation of Professor Morley, he had a variety of other interests and diversions. He was always a critical, but charitable, observer of the contemporary scene, more inclined to hold fast to what he thought good in the older order than to become enthusiastic over new departures. He had an unusual sense of proper balance. He liked to travel, but not too far. He liked to walk, and even to take walking trips of five or six days, but he also liked to linger for a time if an unusually attractive stopping place appeared. He enjoyed music, and was for a number of years a member of the Baltimore Choral Society, but he probably enjoyed most the lighter forms of music. Though he did some things well, he never confessed to any form of skill. I knew that he played chess on occasions, but was surprised to learn recently that he was sufficiently expert to have once beaten Lasker at a time when Lasker was world champion. This interesting fact he had not seemed to think worth mentioning.

Those who were privileged to know him well will never forget the kindly aspects of his daily life nor the devotion with which he pursued the high cultural and scientific ideals which he had set. His personal achievements and his stimulating influence on the mathematics of his time assure him a permanent place in the history of the science.

ARTHUR B. COBLE

CARNAP ON LOGICAL SYNTAX

The Logical Syntax of Language. By Rudolf Carnap. (International Library of Psychology, Philosophy, and Scientific Method.) Translated from the German by Amethe Smeaton (Countess von Zeppelin). New York, Harcourt, Brace, 1937. 16 +352 pp.

The logical syntax of a symbolic language is a study of the formal properties of sentences of that language. It includes the *formation rules* which determine how the symbols of the language can be combined to form sentences, the *transformation rules* which specify when one sentence of the language can be deduced from other sentences, and the other properties of and relations between sentences which can be defined on the basis of these rules. Syntax is a combinatory analysis of expressions, that is, of finite ordered series of symbols. Hence syntax never refers to the meaning of these symbols. Hilbert showed that a clear, formal presentation of the foundations of mathematics must use a metamathematics which is really a syntax of mathematics. The notions of syntax are of central importance for the current growth of mathematical logic.

The present book systematically develops the concepts of syntax, first for two specific Languages I and II, then for an arbitrary language. The specific Language I is a definite ("constructivist" or "finitist") language. It contains the usual calculus of propositions (not, and, implies, $\cdots$) and a Peano arithmetic, with a symbol for 0 and for successor, and with the customary axioms. Variables representing numbers are included, but the quantifiers like "there exists an x" occur only in a limited form, such as "$(\exists x)3(P(x))$," meaning "there exists an x with $x \leqq 3$ such that $P(x)$," and "$(Kx)5(Q(x))$," denoting the smallest $x \leqq 5$ with the property Q.

Language II is a much richer language, and contains everything usually included in a symbolic logic: all of Language I, plus variables for sentences (that is, propositions), variables for predicates, and variables for functors. Such "functors" are functions with any number of arguments of any type. Quantifiers "there exists an x" and "for all x" are used with all these variables. The predicates, which serve also as classes, are classified by the usual (unbranched) type theory, so that a class of numbers is of lower type than a class of classes of numbers. The language so obtained is of interest because it strives for a maximum of flexibility and not, as is often the case, for a minimum of primitive ideas.

Such symbolic languages are ordinarily restricted to symbols defined by means of the primitive symbols of logic and mathematics. Here, in order to make clearer the nature of language and to prepare for a subsequent discussion of the language of science, Carnap allows Languages I and II to contain not only predicates defined in logical terms, but also *descriptive* predicates and functors. One such descriptive symbol is the temperature functor "*te*," which is to be used so that "$te(3)=5$" means "the temperature at the position 3 is 5." Carnap contends that all sentences of physics can be similarly rendered by a "coordinate" language in which the basic symbols are numbers and not names. The general contention seems to neglect the necessity of specifying by name the coordinate system and the scale of measurement to be used.

The syntax of Languages I and II includes the definitions of such important terms as "directly derivable," "demonstrable," and "refutable." In Language I, the specifications under which one sentence is directly derivable from other sentences include the usual rule, that "A_2" and "A_2 implies A_3" give "A_3", in the following form: If the

sentence S_1 consists of a partial sentence S_2 followed by an implication symbol followed by a partial sentence S_3, then S_3 is directly derivable from S_1 and S_2. A *derivation* is a finite series of sentences, such that every sentence of the series is either one of the primitive sentences, or a definition-sentence, or is directly derivable from sentences which precede it in the series. A sentence S is *demonstrable* if there is a derivation in which S is the final sentence. A sentence S is *refutable* if each free variable-symbol in S can be replaced throughout S by a constant number-symbol in such a way that the negate of the resulting sentence is demonstrable. These definitions of syntactical terms may indicate how syntax has to do only with the order and arrangement of symbols into expressions, sentences, and groups of sentences.

These syntactical terms all have to do with an enumerable set of objects, the expressions of the language. If a fixed correlation of these expressions to the natural numbers is chosen, then each syntactical property of expressions becomes a property of the corresponding natural numbers, and can usually be defined by a recursive definition. But natural numbers and recursive definitions can be formulated within the symbolism of Language I (or II). Hence the syntax of either language can be arithmetically formulated within that language. This arithmetized syntax, due to Gödel, makes possible the construction of an arithmetic sentence which, syntactically interpreted, asserts its own indemonstrability. If the language is consistent,* this sentence can be neither demonstrable nor refutable. This discussion in English of Gödel's theorem and its striking consequences should prove valuable to many readers.

Logical positivists formerly distinguished between logic (including mathematics) and empirical science, on the ground that the sentences of logic are always resoluble (either demonstrable or refutable), while sentences of science need not be resoluble (on the basis of logical rules). Gödel's construction of a mathematical sentence which is neither demonstrable nor refutable made this distinction untenable. Apparently in order to reintroduce the distinction, Carnap defines a class of *analytic* sentences, wider than the class of demonstrable sentences. A class of contradictory sentences is also defined, the fundamental result being the theorem that every sentence built up out of logical symbols only is either analytic or contradictory. The definition of "analytic" in Language II is involved, since it includes an (apparently extraneous) reduction process due to the Hilbert school. The more essential features can be illustrated by the (demonstrable) sentence

$$(F)\ (\exists x)\ (F(0) \vee \sim F(x)).$$

By the definition, "this sentence is analytic" can be shown to mean† "for every class B of number symbols there is at least one number symbol such that either this number symbol does not belong to B, or else the symbol 0 belongs to B." The latter sentence seems to be practically a translation of the given sentence into the auxiliary language which is being used for syntax. Similar results hold for other sentences. Thus, to prove that the principle of mathematical induction in Language II is analytic, Carnap must assume the same principle in the syntax language. The statement that a certain sentence is analytic amounts essentially to a careful statement, in the syn-

* Gödel's proof requires also that the language be "ω-consistent." Carnap's discussion slurs over this point. J. B. Rosser has since shown how the assumption of ω-consistency can be avoided, *Extensions of some theorems of Gödel and Church*, Journal of Symbolic Logic, vol. 1, p. 87.

† The definition itself does not use the "either · · · or" in the syntax language, but instead the usual truth value table.

tax language, of the usual "meaning" of the sentence. Furthermore, the definition of "analytic in language S" would seem to require a syntax language at least as strong as the language S being studied. The utility of this notion of "analytic" thus might be open to doubt, especially as Carnap has adduced no positive evidence of the impossibility of a construction according to Gödel of a sentence asserting its own non-analyticity.

Carnap next turns to the difficult and far reaching subject of general syntax. This is a syntactical investigation of any symbolic language whatever. The methods used are essentially those of abstract mathematics. Of the language it is assumed only that there are rules of formation and rules of direct consequence. The latter rules specify when a sentence S is a direct consequence of a class K of sentences. In the special case when the class K is finite, we have a rule of inference of the usual sort. On this narrow basis Carnap succeeds in defining many different general syntactical terms applying to the language: variable, constant, universal operator, arithmetic in a language, predicate, functor, translation into another language, the level of a symbol (as in the theory of types), and the like.

Variables, it has long been recognized, are not variable *things* of some mysterious sort; they are rather symbols for which, under certain circumstances, various other symbols, called "constants," may be substituted. Carnap gives a detailed analysis of this situation, defining such terms as "variable expression," "open expression," "variable," and "constant" in any language.* Here an expression is "open" if it contains at least one free variable. Following the definition of variable expression in Language I, we note that if the symbol "0" in the sentence "0=0" be replaced throughout by any other symbol for a constant number, the result is still a demonstrable sentence. According to the definition given, this fact apparently makes the symbol "0" a variable expression.† Furthermore, the sentence "0=0" turns out to be an open sentence, contrary to Carnap's previous usage in Language I.

To determine, by the definition, whether a symbol is a variable, one must know all the other variables of the language. This is because one cannot substitute an expression for a variable if the expression contains some other variable which would become bound (governed by a quantifier) after the substitution. This would indicate that Carnap's definition does not define the phrase, "this symbol is a variable in the given language." It defines rather "this class of symbols can be considered as a class of variables." There might well be many such classes of variables in a language, and in this event the term "variable" and other terms defined from it would have no fixed meaning.

Another fundamental concept of general syntax is that of the "logical" sentences of a language S. In Language I and II Carnap classes the usual primitive symbols of logic and mathematics, plus all symbols defined exclusively in terms of these primitive symbols, as *logical* symbols. In general syntax this classification by enumeration is to be replaced by a criterion based on the theorem that every logical sentence is determinate, that is, is either analytic or contradictory. The following definition is offered. "Let K_1 be the product of all expressional classes K_i of S, which fulfill the following four conditions. (1) If A_1 belongs to K_i, then A_1 is not empty and there exists a sentence which can be subdivided into partial expressions in such a way that all belong to K_i and one of them is A_1. (2) Every sentence which can be thus subdivided into expres-

* The definitions cannot be significant for every language, for H. B. Curry has developed a language, "combinatory logic," which contains no ordinary variables.

† The contrary assertion is made without proof on p. 195.

sions of K_i is determinate. (3) The expressions of K_i are as small as possible, that is to say, no expression belongs to K_i which can be subdivided into several expressions of K_i. (4) K_i is as comprehensive as possible, that is to say, it is not a proper sub-class of a class which fulfills both (1) and (2). An *expression* is called *logical* if it is capable of being subdivided into expressions of K_1; otherwise it is called *descriptive.*"

The reviewer fails to understand the rôle of condition (4). For suppose that a class K_i contains some expression A_1 which is not a sentence. By the first condition, A_1 is then contained in a sentence S_1. According to condition (4), S_1 must be added to the class K_i, although by condition (3), S_1 cannot be added to K_i. This conflict between conditions (3) and (4) could be avoided by requiring in (4) that K_i is not a proper subclass of a class satisfying (1), (2), and (3) (not merely (1) and (2)). The so modified definition still would not seem to agree with previous usage. For in Language I let K_2 be the class of all logical symbols in the usual sense, while K_3 is the class consisting of all expressions of the form "$(\exists x)$" and all logical symbols, except the existence-operator "$\exists$." These classes satisfy conditions (1) to (3). If necessary, they can be extended to larger classes K_2' and K_3', respectively, which also satisfy (4). Then K_3' cannot contain the symbol "$\exists$," while K_2' cannot contain the symbol "$(\exists x)$," so that K_1, which is part of the intersection of K_2' and K_3', can contain neither "$\exists$" nor "$(\exists x)$." It follows that sentences containing "$\exists$", and logical in the usual sense could not be logical according to this general definition.

This difficulty arises because condition (3) fails to have its intended effect upon the compound expression "$(\exists x)$." The definition of "logical" might be naturally modified as follows: Consider those classes K_i which satisfy (1) and (2) and are maximal with respect to these conditions. For each class K_i denote by L_i the class of those expressions of K_i which cannot be subdivided into several expressions of K_i, and let K_1 be the intersection of all L_i. This avoids the previous difficulty, only to meet another. For in Language I, consider a descriptive functor f (in ordinary usage, f is an empirically defined function $y=f(x)$, where y and x represent integers). Let the class K_4 contain the expressions "$f(0)$," "$=$," and "$\sim$," and all sentences constructed from these expressions. The only such sentences are

$$f(0) = f(0),\ \sim [f(0) = f(0)],\ \sim\sim [f(0) = f(0)],\ \cdots.$$

As "$\sim$" is the symbol for negation, all these sentences are either demonstrable or refutable, and hence are determinate. Thus K_4 satisfies (1) and (2), and so can be embedded in a maximal class satisfying (1) and (2). No numeral, such as 3, can be contained in this class, for "$f(0)=3$" is not a determinate sentence. Hence numerals are not logical symbols under the modified definition, contrary to the usage in Language I. Could the definition of "logical" symbols be further modified to avoid such difficulties?

Such technical points might raise doubts as to the philosophical thesis Carnap wishes to establish here: that in any language whatsoever one can find a uniquely defined "logical" part of the language, and that "logic" and "science" can be clearly distinguished.

Many of the other ingeniously defined concepts of Carnap's general syntax are free from objection. However, the points discussed above show how difficult is the task of defining so many relatively specific concepts in an absolutely arbitrary language. The notion of "any language" may be just as treacherous as was the notion of "any curve" before the critique of analysis situs. Might it not be possible to develop the concepts of general syntax in a more postulational manner? Thus, one might postulate that in the language there are certain symbols, designated as "logical" (or

as "variables"), satisfying certain conditions analogous to those used in the definitions discussed above. Such an approach would recognize the obvious fact that such general syntax, though formulated for any language, is relevant chiefly for languages of the same general type as the Whitehead-Russell calculus, and it would make possible rigorous proofs that the terms defined do in fact agree with corresponding terms as applied to special languages. If such a postulational approach were possible, it would follow the general lines indicated by Carnap's far-reaching and pioneering investigations.

Many confusions and misunderstandings in logic, mathematics, and philosophy can be cleared up, as Carnap shows, by an understanding of the nature and possibilities of syntax. The notion of strict implication, as used in the Lewis logic of modalities, yields one such instance. Usually "strict implication" must be defined in terms of "necessity," so that the postulates must be chosen as the natural properties, if any, of this abstruse and perhaps fuzzy notion. But "*A* strictly implies *B*" can be translated into the clean-cut syntactical statement, "The sentence '*B*' is a logical consequence of the sentence '*A*,' according to the rules of such and such a language." With this reformulation we can now unambiguously determine the properties of strict implication. We also recognize that these properties depend on the language concerned. In the same fashion, any special "logic of modalities" could be replaced by a syntactical translation. Carnap does not assert that it must be so replaced; he follows here and elsewhere a principle of tolerance in syntax: "It is not our business to set up prohibitions, but to arrive at conventions."

Throughout the book, Carnap makes a meticulous and clear-cut distinction between symbols and designations of symbols. The sentences "ω is an ordinal type" and "ω is a letter of the alphabet" appear to have the same subject. Actually, the first sentence is about the object denoted by "ω," the second about the symbol "ω," which is thus to be written in quotations. In this case the distinction is not essential, but in studying syntax it is requisite, for the sentences of syntax are precisely those which speak about symbols. For instance, to say that a sequence is calculable is to make a syntactical assertion about the sequence. Hence it must always be calculable with reference to a certain language.

In philosophical discussion it is important to recognize pseudo-syntactical sentences which do not appear to belong to syntax, but which can be translated into syntax. Carnap, in the last section of this book, shows how many fake problems and misunderstandings can be cleared up by such an analysis of sentences. For instance, "time is continuous" can be translated as "the real number expressions are used as time coordinates." "The world is a totality of facts, not of things" becomes "Science is a system of sentences, not of names." Some of his philosophical distinctions, such as that (p. 289) between the meaning of an expression and the object designated by an expression, are essentially dependent on the definition of "logical" sentences analyzed above. Such philosophical distinctions may therefore be untenable. The book ends with an eloquent discussion of two related theses: Any philosophy is either meaningless or is simply the logic of science; the logic of science is the syntax of the language of science.

The book contains many other illuminating discussions of various aspects of symbolic logic. In particular, we find an extraordinarily general statement of Gödel's theorem for an arbitrary language (unfortunately no proof and no reference to any printed proof is given); a discussion of various famous antinomies, syntactical and otherwise; a discussion of a paradox in certain axiomatic set-theories, according to which all sets are denumerable—syntactically denumerable, but not denumerable

within the language of the set-theory itself. These three questions, and some of the other topics, were not included in the German edition of the book. As a whole, the book is a stimulating and fruitful discussion of syntax, a subject not yet in a definitive form but even now having a wide range of application in mathematics, science, and philosophy.

The following minor corrections might be noted. On page 40, Theorem 14.3 cannot be directly proven by induction. One must rather prove by induction that every logical sentence with n distinct free variables either is contradictory or is analytic with not more than n uses of the non-finite rule DC 2. On page 104, RR 9, read "unlimited operators" for "unlimited sentential operators." On page 21 replace the definition of an open expression by "If a variable which is free at some position in A_1 occurs in A_1 at that position, then A_1 is called open."

SAUNDERS MACLANE

SHORTER NOTICES

Höhere Mathematik für Mathematiker, Physiker, und Ingenieure. By R. Rothe. Teil IV: *Übungsaufgaben mit Lösungen. Formelsammlung.* Edited by O. Degosang. Heft 4: *Unendliche Reihen. Vektorrechnung nebst Anwendungen.* Heft 5: *Raumkurven und Flachen, Linienintegrale und Mehrfache Integrale.* Heft 6: *Gewöhnliche und Partielle Differentialgleichungen nebst Anwendungen.* Leipzig and Berlin, Teubner, 1937. 161 pp.

The previous parts of this volume of exercises have been noted in this Bulletin, vol. 39 (1933), p. 492; vol. 40 (1934), p. 202; and vol. 43 (1937), p. 12. The fourth part contains exercises in infinite series, integrals depending upon a parameter, determinants, and elementary vector analysis. The type of exercise, arrangement, and character is similar to that of the preceding volumes.

Parts 5 and 6 bring to a close the parts of the fourth volume of this work, giving an exercise collection for the preceding volumes. They present a list of interesting exercises on the topics mentioned, as well as their solutions. For the teacher of calculus, these parts contain suggestive exercise material. Worth mentioning in Heft 5 might be the exercises giving practice in the evaluation of line integrals and the determination of the volume elements for a variety of sets of coordinate surfaces. Noticeable in Heft 6 is the comparative absence of applications to other than mathematical fields.

Taking this exercise collection as a whole, we find that it contains much which is ordinary and much which is different and suggestive. The solutions given are for the most part excellent, but for the inquiring student a little more emphasis on the fact that other simple solutions exist might prove worth while.

T. H. HILDEBRANDT

Postulates for Assertion, Conjunction, Negation, and Equality. By Edward V. Huntington. Proceedings of the American Academy of Arts and Sciences, vol. 72, no. 1, 1937, pp. 1–44.

The author gives a set of postulates for an abstract mathematical system which has a certain formal correspondence with Lewis' system of strict implication. Consequences of the postulates are developed in some detail, with proofs given in full, so that the paper is in this way entirely self-contained. The consistency of the system, the independence of the postulates, and a number of additional propositions of independence are established by an interesting series of examples.

Under the possible interpretations of the abstract system which are indicated in the paper, the elements of the system become either (1) a finite set of integers or (2) the sentences of an unspecified language L. In case (1) the system itself becomes a kind of finite algebra; in case (2) it becomes a branch of the syntax of L. It is an interpretation of this second kind which the author declares himself to have chiefly in mind.

The usual postulational method, which is here employed, presupposes a fully developed logic in terms of which the consequences of the postulates are derived; hence it may not without circularity be used in the development of logic itself. For this reason the author's system is not interpretable as a logic, and his comparisons with the propositional calculus of *Principia Mathematica* and with the calculus of strict implication as developed in Lewis and Langford's *Symbolic Logic* are to this

extent erroneous. So far, however, as the form of the development may be taken as implying an opinion that Lewis's theory of strict implication is from the point of view of intuitive justification better formulated as a syntax than as a logic, the reviewer finds himself in complete agreement with the author.

ALONZO CHURCH

Introduction to the Theory of Groups of Finite Order. By Robert D. Carmichael. Boston, Ginn and Co., 1937. 16+447 pp.

The fundamental place occupied by the group concept in modern mathematics and the discovery of new and important applications of the theory of groups have made it necessary for every serious student of mathematics to acquire a working knowledge of this subject. Professor Carmichael's book will find a large audience, as it is addressed to the beginner, not to specialists, and makes no claim to including the most recent discoveries in group theory. The exposition is excellent throughout. The book includes a large number of exercises of varying degrees of difficulty which will enable the learner to cultivate the subject as assiduously as he desires.

The first part of the book treats those topics which are indispensable to all works on the theory of groups: abstract groups, abelian groups, prime-power groups, permutation groups, linear groups in the field of complex numbers, and the theory of group characters. A full discussion of finite fields is followed by a treatment of linear groups in a finite field with emphasis on certain multiply transitive permutation groups whose existence is established most readily by means of linear groups. The author calls attention to certain doubly transitive groups overlooked by Burnside.

Geometers will be particularly interested in the chapters on finite geometries which contain an exposition, from the group standpoint, of the researches of Veblen and Bussey. The chapter on algebras of doubly transitive groups includes Dickson's researches on finite algebras and will appeal to mathematicians interested in linear algebras. The final chapter is devoted to tactical configurations. The latter part of the book is particularly valuable, as it includes material widely scattered throughout the literature and here collected and systematized for the first time.

LOUIS WEISNER

Höhere Algebra. II. *Gleichungen höheren Grades.* By Helmut Hasse. Zweite verbesserte Auflage. (Sammlung Göschen, vol. 932.) Berlin, de Gruyter, 1937. 158 pp.

The new edition is in the main the same as the first edition, giving the principal results of the Galois equation theory. The results are, however, derived under somewhat more general conditions, since the author has dropped the former limitation to perfect fields and considers separable extensions in general. There are also various minor additions and improvements. A short account of the theory of finite fields is added.

OYSTEIN ORE

An Introduction to Projective Geometry. By C. W. O'Hara and D. R. Ward. Oxford, Clarendon Press, 1937. 298 pp.

The point of view of the authors is best stated in the preface: "It cannot with truth be said that the book has been written to 'supply a long-felt want.' There seems, unfortunately, to be very little demand for the teaching of Projective Geometry in

this country. In default of this excuse, therefore, the authors must fall back on another, namely, the hope that their work may do something to stimulate a demand for more wide-spread familiarity with the subject."

In the United States we have been more fortunate; during recent decades our literature has been enriched by several meritorious contributions, from various points of view. Without distinguishing between the rôles of the owl and the egg, we can also mention an eager and general demand for instruction in the subject. The reviewer is of the opinion that there is place for the book under review, both for its distinctive point of view and for the excellence of its presentation.

After a brief historical and critical introduction, the work begins with point and lines as undefined elements, and develops a logical structure in terms of incidence, without appeal to intuition. The notation is particularly well chosen, and generous lists of exercises serve as a constant check. In this first half of the book, the synthetic geometry of one and two dimensions is carried to include Pascal's theorem and poles and polars. On the whole this is well done; however, in their eagerness to avoid concepts in more than two dimensions, the authors give only an algebraic proof of Desargues' theorem on perspective triangles, and this before the good treatment of the algebraic method is started. A similar objection can be made concerning the diagonal points of a quadrilateral.

It is true that these digressions are put in fine print, and are frankly stated to be verifications only, but the context rather helps the reader to infer that they furnish more rigorous proofs. In the opinion of the reviewer the merits of the book would have been decidedly increased by leaving them out altogether until the algebraic foundations had been laid.

The second half is algebraic. It begins with the non-homogeneous mesh-gauge and develops the number system to apply to open sets, with a meaning attached to sum and product, but keeping clearly in mind that one line is not included in the scheme. The next chapter, homogeneous mesh-gauges, removes this restriction for purely descriptive properties and completes what is usually included under the title of a first course in projective geometry, both synthetic and algebraic.

The next chapter, the metric gauge, gives a satisfactory introduction to the use of metrical properties in terms of a non-singular conic, and emphasizes the restriction that elements on the metric gauge (the absolute) do not have metric properties. This is then applied to a singular conic of rank one as a point conic and rank two as line conic.

A short chapter on ternary linear transformations gives a meaning of collineations and correlations, without discussing the configurations of invariant elements. This is followed by a still shorter chapter on applications to physics, including special relativity.

The mechanical make-up of the book deserves high praise. The type is clear, the printing well done, the figures excellent, and the proofreading faultless. Dates are ordinarily associated with publication; in the case of Desargues and of Pascal that of birth is given, but the date associated with Pappus (200 B.C.) seems odd.

The reviewer hopes that the *Introduction to Projective Geometry* by O'Hara and Ward may be widely read.

VIRGIL SNYDER

NOTES

During recent years the Department of Mathematics of the University of Chicago has planned to place emphasis on a particular field each Summer Quarter. Analysis was stressed in the summers of 1935 and 1937, and algebra will be emphasized in 1938. This is a field of major interest of Professors Dickson, Albert, and Hull, as well as Drs. MacLane, Jacobson, and Dribin, all of whom will participate in a seminar on the subject. There will also be a Conference on Algebra beginning at 9:30 A.M. on Tuesday, June 28, and ending July 1. During this period the participants in the seminar and all other mathematicians who may be interested will be addressed by the following invited speakers (listed in the order of their appearance on the program and with the titles of their addresses): L. E. Dickson (Chicago), *On the history and nature of the integral quantities of algebras*; Ralph Hull (Illinois), *The arithmetic of rational cyclic algebras*; A. A. Albert (Chicago), *On cyclic algebras*; Richard Brauer (Toronto), *On the structure of normal division algebras*; Saunders MacLane (Chicago), *The structure of perfect fields*; Oscar Zariski (Johns Hopkins), *Ideal theory and algebraic geometry*; Solomon Lefschetz (Princeton), *Formal power series in algebraic geometry*; C. C. MacDuffee (Wisconsin and the Institute for Advanced Study), *The use of matrices in ideal theory*; C. G. Latimer (Kentucky), *Ideals in a quaternion ring and Hermitian forms*; Emil Artin (Notre Dame), *Theory of quadratic forms*; Reinhold Baer (North Carolina), *Group structure and subgroup lattices*; John Williamson (Johns Hopkins), *The conjunctive equivalence of pencils of Hermitian and anti-Hermitian matrices*; M. H. Ingraham (Wisconsin), *Topics in the theory of matrices whose elements belong to a division algebra*; O. F. G. Schilling (Johns Hopkins), *Local class field theory*; Nathan Jacobson (North Carolina), *Simple Lie algebras*. It is hoped that many mathematicians, especially those interested in abstract algebra, will avail themselves of this opportunity to come together. Housing accommodations near the University have been arranged for those attending the Conference. A limited number of rooms will be available in Judson Court, a conveniently located and modern residence hall facing the Midway Plaisance, at the moderate charge of $2.75 per day for room and board. Those desiring to reserve accommodations may do so by writing to A. A. Albert, Eckhart Hall, The University of Chicago.

The symposium on the algebra of geometry and related subjects at the University of Notre Dame, announced in the January number of this Bulletin, was held February 11 and 12, in five sessions. At the first session, on Friday morning, the following papers were presented: *Boolean algebra and topology*, by Professor M. H. Stone; *Partially-ordered function spaces*, by Mr. Garrett Birkhoff. At the session on Friday afternoon the following papers were presented: *On structures and their applications*, by Professor Oystein Ore; *Applications of the lattice theory to field structure*, by Dr. Saunders MacLane; *Some problems concerning groups*, by Dr. J. K. Senior. On Friday evening Professor E. V. Huntington gave a general lecture on the method of postulates entitled *The duplicity of logic*. On Saturday morning the following papers were presented: *Continuous geometry*, by Professor John von Neumann; *Alt's algebra of affine geometry and related problems*, by Professor Karl Menger; *A theorem on the algebra of geometry*, by Dr. A. N. Milgram. At the last session, on Saturday afternoon, the following papers were presented: *An application of the theory of division algebras*

to geometry, by Professor A. A. Albert; *An algebraic topic with geometric aspects*, by Professor Emil Artin.

Professor E. A. Milne has been elected president of the London Mathematical Society. Professor G. B. Jeffery, Miss M. L. Cartwright, and Mr. J. Hodgkinson have been elected vice presidents.

At the recent meeting of the American Association for the Advancement of Science held at Indianapolis, Professor J. R. Kline was elected vice president of the Association and Chairman of Section A for the year 1938.

Professor M. H. Ingraham, of the University of Wisconsin, has been elected president of the American Association of University Professors.

The Paris Academy of Sciences announces the award of the following prizes for 1937: the Francoeur prize to Jean Leray, for his work on mathematical analysis; the Montyon prize in mechanics to Lucien Malavard, for his electric apparatus permitting the rapid calculation of hydrodynamic circulations; the Poncelet prize to Joseph Bethenod, for his work on mechanics and electricity; the Pierson-Perrin prize to Léon Brillouin, for his work as a whole; the Lalande prize in astronomy to Michel Giacobini, for his work on stellar astronomy and on the comets; the Benjamin Walz prize to Maurice Burgaud, for his work in China on terrestrial magnetism; the G. de Pontecoulant prize to Henri Roure, for his work on planetary perturbations; the Binoux prize to Raoul Goudey and Jean Lagrula, for their gravimetric observations in France and northern Africa; the Henri de Parville prize in physics to Paul Lainé, for his research on the magnetic and magneto-optic properties of liquified gases; the Kastner-Boursault prize to Bernard Decaux, for his work on the applications of quartz to chronometry; the Gaston Planté prize to Jean Bernamont, for his studies on the fluctuations of electric resistances; the François Hebert prize to Paul Couderc, for his works on astronomy, especially that entitled *Univers*, 1937; the Hughes prize to Paul Soleillet, for his work on the structure of light; the Clément-Félix foundation prize to Étienne Vassy, for his work on the atmospheric ozone; Montyon prizes in statistics to Mlle. Dagmare Weinberg, for her important contribution to the work *La correction des épreuves écrites dan les examens*; to Adolphe Lesage and Marcel Moine, for their study of infant mortality; and to Marcel Luc-Verbon, for his studies on the science of theoretical values; the Bordin prize to Georges Darmois, for his work on the calculus of probabilities; the Petit D'Ormoy prize to Paul Lévy, for his mathematical work as a whole; the Saintour prize to André Marchaud, for his work on geometry; an award from the Hirn foundation to Marcel Fouché, for his work on acoustics; an award from the Henri Becquerel foundation to André Savornin, for his work on terrestrial magnetism; an award from the Loutreuil foundation to Abbé Zephirin Carrière, for the purchase of apparatus for the analysis of sounds; and awards from the Villemot foundation to Mme. Rose Bonnet-Sainturier, for the publication of a memoir on the double stars; to Aimé Cotton, for the laboratory of the electromagnet of the Academy of Sciences; to Alexandre Dauvillier, for the construction and installation of a recording electrometer; and to Charles Maurain, for his researches relative to terrestrial magnetism and atmospheric electricity.

The Academy of Science of Turin has conferred on Professor Leonida Tonelli the Bressa prize for 1937.

Professor L. Berzolari, of the University of Pavia, and Professor G. Scorza, of the

University of Rome, have been elected national associates of the Royal Academy of Sciences of Turin. Professor U. Cisotti, of the University of Milan, Professors A. Terracini and F. Tricomi, of the University of Turin, have been elected corresponding members.

Professor Enrico Fermi has been elected to membership in the Academy of Science of Vienna.

Professor Tullio Levi-Civita has been elected to membership in the Brazilian Academy of Science and has been awarded an honorary doctor's degree by the University of San Marco of Lima.

Professor Mauro Picone has been named honorary member of the Academy of Science of Roumania.

Professor G. Scorza, of the University of Rome, has been elected a national member of the Royal Academy of the Lincei.

Professor Leonida Tonelli has been appointed a member of the committee of directors of the Italian National Research Council.

Dr. D. C. Murdoch, of the University of Toronto, has been awarded a Sterling Fellowship at Yale University.

Professor J. A. Shohat has been elected Fellow of the American Institute of Statistics.

Professor Edoardo Amaldi has been appointed professor of experimental physics at the University of Rome.

Professor Vincenzo Amato, of the University of Cagliari, has been appointed professor of mathematical analysis at the University of Messina.

Professor Gilberto Bernardini has been appointed professor of experimental physics at the University of Camerino.

Professor Luigi Campedelli, of the University of Cagliari, has been appointed professor of projective and analytic geometry at the University of Florence.

Professor E. J. Cartan, of the University of Paris, has been appointed Rouse Ball Lecturer at the University of Cambridge for the year 1937–1938.

Professor Giovanni Gentile has been appointed professor of theoretical physics at the University of Milan.

Professor Ettore Majorana has been appointed professor of experimental physics at the University of Naples.

Professor Basilio Mania has been appointed professor of mathematical analysis at the University of Pavia.

Professor Arturo Maroni, of the University of Modena, has been appointed professor of analytic and projective geometry at the University of Pavia.

Professor Carlo Miranda has been appointed professor of mathematical analysis at the University of Genoa.

Professor Guilio Racah has been appointed professor of theoretical physics at the University of Pisa.

Professor Ivo Ranzi has been appointed professor of experimental physics at the University of Cagliari.

Professor G. C. Wick has been appointed professor of theoretical physics at the University of Palermo.

In further explanation of the note which appeared at the top of page 762 in the November, 1937, issue of this Bulletin, it should be stated that Professor Emil Artin has joined the faculty of Notre Dame and that Abbé Lemaître will be visiting professor during the spring term.

Assistant Professor R. G. Archibald, of Columbia University, has been appointed to an assistant professorship at Queens College, Flushing, N. Y.

Assistant Professor A. E. Brandt, of the Iowa State College, is on leave of absence this year. He is serving as senior statistician at the United States Bureau of Soils, Department of Agriculture.

Assistant Professor A. B. Brown, of Columbia University, has been appointed to an assistant professorship at Queens College, Flushing, N. Y.

Assistant Professor S. S. Cairns, of Lehigh University, has been appointed to an assistant professorship at Queens College, Flushing, N. Y.

Assistant Professor A. T. Craig, of the University of Iowa, has been promoted to an associate professorship.

Dr. D. B. DeLury has been appointed lecturer in mathematics at the University of Toronto.

Professor Peter Field, of the University of Michigan, has leave of absence for the second semester of 1937–1938.

Dr. C. H. Fischer, of Wayne University, has been promoted to an assistant professorship.

Associate Professor Philip Franklin, of the Massachusetts Institute of Technology, has been promoted to a professorship.

Dr. Evan Johnson, of the Uniontown Extension Center of the Pennsylvania State College, has been promoted to an assistant professorship.

Dr. C. W. MacGregor, of the Massachusetts Institute of Technology, has been promoted to an assistant professorship.

Professor Karl Menger, of the University of Notre Dame, has been made chairman of the department of mathematics.

Dr. E. R. Ott, of the University of Buffalo, has been promoted to an assistant professorship.

Dr. E. L. Post, of the College of the City of New York, has been promoted to an assistant professorship.

Mr. W. O. Rogers, of Pennsylvania State College, has been promoted to an assistant professorship.

Assistant Professor Wladimir Seidel, of the University of Rochester, has been granted leave of absence for the academic year. He is spending the year at Brown University.

Mrs. O. H. Stecker, of the Pennsylvania State College, has been promoted to an assistant professorship.

Assistant Professor J. M. West, of the Pennsylvania State College, has been promoted to an associate professorship.

The following appointments to instructorships have been announced: University of Alabama: Mr. L. D. Rodabaugh; George Washington University: Dr. W. C. Mitchell; Montana State College: Dr. Nathan Schwid; University of Pittsburgh: Mr. J. O. Blumberg; Queen's College: Dr. M. M. Sullivan; South Dakota State College: Dr. L. E. Mehlenbacher; University of Texas: Mr. E. L. Godfrey, Miss H. C. Miller, Mr. R. H. Sorgenfrey, Mr. R. L. Swain; Wayne University: Dr. T. H. Southard, Dr. M. C. Wolf.

Dr. L. N. G. Filon, Goldsmid professor of mathematics at the University of London and formerly vice chancellor of the University, died on December 29, 1937, at the age of sixty-two years.

Dr. A. Lodge, formerly professor of pure mathematics at the Royal Indian Engineering College and president of the Mathematical Association (English) in 1897–1898, died recently at the age of eighty-three years.

Professor J. D. Bond, of the University of Tennessee, died on November 10, 1937.

Associate Professor Orpha E. Worden, of Wayne University, died at Miami Beach, Florida, on February 7, 1938.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

90. R. P. Agnew: *Cores of complex sequences and of their transforms.*

It is shown that a regular transformation $\sigma_n=\sum_{k=1}^{\infty}a_{nk}s_k$, determined by a matrix a_{nk} of complex constants, transforms each definitely divergent sequence s_n (K. Knopp, Mathematische Zeitschrift, vol. 31, pp. 97–127) into a definitely divergent sequence σ_n if and only if there is an index K such that a_{nk} is real and non-negative for all $k\geqq K$. This condition is also necessary and sufficient to ensure that the core ("Kern" of Knopp, loc. cit.) of each sequence s_n contain the core of its transform σ_n. Some related results are obtained, including a characterization different from but equivalent to Knopp's definition of definite divergence. Finally a theorem of Steinhaus type (Prace Matematyczno-fizyczne, vol. 22, pp. 121–134) is given with comments relevant to cores. (Received January 17, 1938.)

91. A. A. Albert: *Non-cyclic algebras with pure maximal sub-fields.*

The author modifies the algebras of his paper in the Transactions of this Society, vol. 35 (1933), pp. 112–121, and thereby proves the existence of normal division algebras of degree four over a non-modular field K which are non-cyclic yet have maximal sub-fields $K(j)$, $j^4=g$ in K. This proves the falsity of a recent conjecture on normal division algebras (see introduction of the paper in the Transactions of this Society, vol. 39 (1936), pp. 183–188). (Received January 22, 1938.)

92. E. F. Beckenbach and M. Reade: *Generalizations to space of the Cauchy and Morera theorems.*

The theorems of Cauchy and Morera are generalized for three-space. A typical result is the following: If $x_j(u,v)$, $(j=1,2,3)$, are harmonic functions in a domain D, then a necessary and sufficient condition that these functions be the coordinate functions of a minimal surface given in isothermic representation is that for each circle C, lying in and enclosing only points of D, $\sum_{j=1}^{3}[\int_C x_j(u,v)\cdot(du+idv)]^2=0$. (Received January 26, 1938.)

93. Garrett Birkhoff: *Dependent probabilities and spaces (L).*

The paper bases the theory of dependent probabilities on the theory of partially ordered function spaces. This formulation makes it possible to include all known cases in a single set of definitions (cf. also abstracts 43-9-320-321). A new theorem, which specializes in the *deterministic* case to von Neumann's mean ergodic theorem, is proved with added generality in the *stochastic* case. (Received January 28, 1938.)

94. Garrett Birkhoff: *Partially ordered linear spaces.*

A "K-space" is defined (Kantorovitch) as a linear space over the real field, which is a lattice, in which $x\to x+a$ and $x\to\lambda x$, $(\lambda>0)$, preserve order, and $x\to -x$ inverts order. The "normal subspaces" corresponding to homomorphisms which preserve inclusion are characterized abstractly; they form a *distributive* lattice, and the components of direct decompositions of L form the Boolean algebra of complemented elements in this lattice. Any "bounded" additive functional λ (that is, one bounded on bounded subsets of L) is the sum of its positive and negative parts; hence, the bounded additive functionals form a "conjugate" K-space. There is just one K-space of each finite dimension n. Define a K-space as "continuous" when every bounded monotone sequence has a limit, $\lambda(x)$ as "continuous" if $x_n\uparrow x$ implies $\lambda(x_n)\to\lambda(x)$. Under these added hypotheses, L is the direct sum of three components—on which λ is negative, zero, and positive, respectively. If λ is positive (that is, if $f>0$ implies $\lambda(f)>0$), then L is a linear metric space relative to $\lambda(|f-g|)$, and a Banach space of type (L) if and only if L is "continuous." (Received January 28, 1938.)

95. R. P. Boas (National Research Fellow) and Salomon Bochner: *Closure theorems for translations.*

Let D denote the space of complex-valued functions $g(t)$ defined on $(-\infty, \infty)$, continuous except for discontinuities of the first kind, and such that $g(\infty)$ and $g(-\infty)$ exist; let C denote the subspace of continuous functions $f(t)$ such that $f(\infty)=f(-\infty)=0$. Every $g(t)$ of D has a generalized Fourier transform $G(t)$ of order 2 (as defined by S. Bochner, *Vorlesungen über Fouriersche Integrale*, chap. 6). It is possible to approximate uniformly on $(-\infty, \infty)$ to any $f(t)$ of C by finite linear combinations of translations, $g(t+u)$, of $g(t)$ of D if and only if $G(t)$ is linear on no interval. The proof depends on theorems from operation theory, Kaltenborn's determination of the general linear functional on D (this Bulletin, vol. 40, p. 702), and theorems used by Wiener in discussing the closure of translations of functions of L. Similar results are obtained when one admits functions $g(t)$ which are $O(|t|^n)$, $(|t|\to\infty;\ n=1, 2, \cdots)$. (Received January 25, 1938.)

96. J. W. Calkin: *Abstract self-adjoint boundary conditions.*

Let L be an arbitrary differential operator applicable to functions defined over an open set E in euclidean n-space and coincident with its formal adjoint. One may then consider the problem of determining, when possible, a lucid definition of the boundary conditions defining those domains in the Hilbert space $\mathfrak{L}_2(E)$ on which L induces self-adjoint transformations. Aside from the matter of closure, this problem involves in large part questions of an algebraic or quasi-algebraic nature. It is a primary function of the present work to provide an abstract algebraic framework within which such questions can be clearly answered, under certain reasonable restrictions on L. The author has found that the results apply to various partial differential operators, among them operators of the form $-\partial(p\partial/\partial x)/\partial x-\partial(p\partial/\partial y)/\partial y+q$ (cf. abstract 43-3-114) and their iterates, as well as to ordinary differential operators. (Results for ordinary differential operators have been obtained directly by M. H. Stone and I. Halperin.) The theory also has other implications; in particular, it contains in essence an independent development of von Neumann's theory of Cayley transforms, motivated quite differently from the original. A brief account of some of the more fundamental results appears in the Proceedings of the National Academy of Sciences, vol. 24 (1938), no. 1. (Received January 28, 1938.)

97. Richard Courant: *The existence of minimal surfaces of least area bounded by prescribed manifolds.*

By means of a general theorem concerning potential functions, the existence proof for minimal surfaces of least area is given if the boundary consists of prescribed Jordan arcs and point sets free to lie on prescribed continuous manifolds of more than one dimension. The topological structure of the minimal surface can be prescribed under certain inequality conditions. (Received January 18, 1938.)

98. H. S. M. Coxeter: *A plane projection of the polytope 2_{21} whose 27 vertices represent the lines on the general cubic surface.*

Among the 27 lines on a cubic surface, three (such as c_{12}, c_{56}, c_{18}), of which two intersect while the third is skew to both, uniquely determine a fourth (c_{25}) which intersects the third but is skew to the first two. This suggests a representation of the 27 lines by 27 points of a plane, the lines of such an "XX configuration" being represented by the vertices of a parallelogram. Pairs of the points are joined or not joined according as they represent skew or intersecting lines. One of the most symmetrical arrangements is obtained by beginning with a regular enneagon, whose vertices represent a cycle of nine lines such as a_1, c_{25}, c_{56}, b_2, c_{16}, b_5, c_{12}, c_{24}, a_6. The rest of the points follow immediately by completing parallelograms; they form two smaller enneagons, concentric with the first. The whole figure can be regarded as an orthogonal projection of the vertices and edges of Gosset's six-dimensional semi-regular polytope 2_{21}, whose relation to the cubic surface was noticed by Schoute and explained by Du Val (Proceedings of the London Mathematical Society, (2), vol. 35 (1932), pp. 23–74). The parallelograms arise as projections of squares. (Received June 29, 1937.)

99. J. J. De Cicco: *Fields whose geodesic series can be represented by the turbines of a flat field.*

This paper is a continuation of the paper, *Fields whose geodesic series have circles for their point- or line-unions* (abstract 43-11-377), by the author. A transformation between two fields is said to be *equi-deviate* if the distance between the centers of the tangent turbines at a common element of any two field series is preserved. In this paper the following two results are proved: 1. In order that a field be mapped into a flat field in such a way that the geodesic series of the field correspond to the turbines (the geodesic series) of the flat field, it is necessary and sufficient that the curvature G of the field be the same for all elements of any equi-parallel series of the field. 2. In order that a field be mapped by an equi-deviate transformation into a flat field in such a way that the geodesic series of the field correspond to the turbines of the flat field, it is necessary and sufficient that the curvature G of the field be constant and less than one. The analogous problem in three-space was discussed by Kasner in the paper, *The generalized Beltrami problem concerning geodesic representation*, Transactions of this Society, vol. 4 (1903), no. 2. (Received January 10, 1938.)

100. Arnold Dresden: *An approach to existence theorems in the calculus of variations.* Preliminary report.

By using the fact that a lower semicontinuous function, defined in a metric space, has extreme values on any self-compact set in that space, the author intends to establish existence theorems for a variety of functionals occurring in the calculus of variations. The problem falls into two parts: (1) the determination of conditions on the

integrand function which insure lower semicontinuity of the functional on the space of absolutely continuous functions; for this part the work of Tonelli, McShane, and others is drawn upon; (2) the setting up of conditions under which the classes of absolutely continuous functions occurring in the various functionals are self-compact. The first part needs to be done only once for all functionals in which the same integral occurs. The second part requires a study of the class of absolutely continuous functions and of successive subclasses. This preliminary communication discusses the general mode of approach and announces results, many of which have been obtained before, some of which are believed to be new. (Received January 27, 1938.)

101. Melvin Dresher and Oystein Ore: *On the theory of multigroups.*

This paper contains a theory of multigroups (hypergroups), that is, group-like systems in which the multiplication is not one valued. The results of Marty and Wall are extended and a general discussion of the structure properties is given. A certain class of multigroups whose composition series consist of ordinary groups is of particular interest. (Received January 25, 1938.)

102. J. E. Eaton: *A formula for the coefficients of the cyclotomic polynomial.*

A formula for the coefficients of the irreducible equation of the primitive nth roots of unity is derived. The formula is valid for any n. (Received February 1, 1938.)

103. Tomlinson Fort: *Theorems on the summability of series.*

Theorems for summability, analogous to the Abel-Dirichlet theorems in the theory of convergent series (see Fort, *Infinite Series*, Theorems 119 and 120), are developed in this paper. Borel summability is first considered and several general theorems, which are applied in detail to power series and Lambert series, are obtained. In the treatment of R-summability, general theorems, which are applied in detail to certain exponential series and to generalized factorial series, are proved. (Received January 26, 1938.)

104. J. S. Frame: *A symmetric representation of the 27 lines on a cubic surface by lines in a finite geometry.*

The group of isomorphisms of the 27 lines and 45 triple-tangent planes on a cubic surface is of order 51840 and contains the simple group of order 25920 as an invariant subgroup. The latter may be represented as a group of unitary transformations in a three-dimensional finite projective geometry, in which a plane is defined by four homogeneous coordinates from a modular field of four marks. A one-to-one correspondence between the 45 triple-tangent planes to the cubic surface and a certain set of 45 planes in the finite geometry is established, providing a simple criterion that two planes meet in a line of the surface. The 27 lines may be represented by those 27 number triples (a, b, c) which have just one coordinate equal to zero. Then the condition that two lines intersect on the surface is that they have just one coordinate in common. The third line of the triangle in which their plane cuts the surface is their vector sum (mod 2). Several configurations of lines and planes on the cubic surface are easily discussed in this representation, which seems, on account of symmetry, to be better adapted to their study than the familiar notation of Schläfli. Such, for instance, are the 40 triples of trihedral pairs, which are closely related to sets of those

40 planes of the finite geometry which do not correspond to triple-tangent planes on the cubic surface. (Received January 28, 1938.)

105. J. W. Givens: *Cayley parametrization and spin representation of orthogonal matrices.*

The spin representation of the complex orthogonal group leads to the association of two sets $\pm(\lambda, \lambda^{ij}, \lambda^{ij,kl}, \cdots)$ of skew symmetric tensors of even rank with each proper orthogonal matrix $L=\|L^i_j\|$. The set contains one scalar and n tensors, where the order of L is $2n$ or $2n+1$. The components of these tensors are parameters which give an unexceptional representation of all proper orthogonal matrices. The relations connecting these tensors are found. One of them is $\lambda(L^i_j-\delta^i_j)=\lambda^{ir}\delta_{rs}(L^s_j+\delta^s_j)$, where $\|\delta_{rs}\|=\|\delta_{sr}\|=1$. Hence $\lambda^{-1}\|\lambda^{ij}\|$ is the Cayley matrix $A=(L-1)(L+1)^{-1}$, which parametrizes all those orthogonal matrices for which $|L+1|\neq 0$. If the order of L is even and $|L-1|\neq 0$, the last two tensors in the set are related to the matrix $(L+1)(L-1)^{-1}$. These results are also obtained for the real orthogonal and Lorentz groups. (Received January 27, 1938.)

106. F. L. Griffin: *Discontinuous solutions for certain problems in the calculus of variations.*

In this note the author constructs, for certain new problems in the calculus of variations, discontinuous solutions analogous to those of Goldschmidt for the minimum area of revolution. Two of the problems considered relate to the surface of revolution which has the minimum moment of inertia with respect to the axis of revolution, or which exerts the minimum attraction upon a particle located on that axis. Continuous solutions of the several problems by extremals have been obtained in thesis studies by former students. (Received January 29, 1938.)

107. Israel Halperin: *Dimensionality in reducible geometries.*

The dimensionality theory of J. von Neumann, which applies to irreducible (classical projective and continuous) geometries, has been extended by the writer to reducible geometries with suitable sets of partial-automorphisms (see abstract 43-3-125). The present paper constructs an extensive class of reducible geometries to which this dimensionality theory applies. This is done by defining integration for functions whose values lie in an arbitrary continuous geometry. (Received January 28, 1938.)

108. Philip Hartman and R. B. Kershner: *On the Fourier transform of a singular function.*

The Fourier-Stieltjes transform $L(T;\sigma)=\int_{-\infty}^{+\infty}\exp(itx)d\sigma(x)$ of any distribution function $\sigma(x)$ is always $O(1)$. A necessary condition for $\sigma(x)$ to be absolutely continuous is that $L(T;\sigma)=o(1)$. Every known example of a distribution function $\sigma(x)$, which is not absolutely continuous but which satisfies $L(T;\sigma)=o(1)$, is almost everywhere constant. The authors give an example of a purely singular distribution function $\tau(x)$, such that the spectrum of τ is an entire interval, $L(T;\tau)=O(\log^{-1/2}|t|)$, and $L(t;\tau)\neq o(\log^{-1/2}|t|)$. The function τ is obtained as the infinite convolution of a sequence of simple step-functions. (Received January 27, 1938.)

109. Max Herzberger: *Partial differential equations and variation calculus.*

It is shown that the theory of partial differential equations and the geometrical

problems in the calculus of variations can be considered as equivalent in many respects. A kind of Legendre transformation is used to transform problems from one region into problems of the other and vice versa. This concept allows the application of the results and methods found in one part to those of the other, especially to all of Hamilton's ideas. Simple geometric interpretations of the integration theories by Cauchy, Lagrange, Mayer, Natani, and Lie are thus obtained. It is also recognized why the theory of integral invariants must play an important rôle in both fields. The relation between the two fields becomes most significant if, in both problems, homogeneous coordinates are used. It is easy to develop the theory for the more complicated non-homogeneous case later on. (Received January 19, 1938.)

110. Edward Kasner and J. J. De Cicco: *The quadric fields in the geometry of the whirl-motion group G_6.*

This paper is a continuation of the paper, *The group of turns and slides and the geometry of turbines*, by Kasner, American Journal of Mathematics, vol. 33 (1911), and the abstract by the authors, *The geometry of the whirl-motion group G_6. Elementary invariants* (abstract 44-1-28). A *turbine* consists of the ∞^1 elements whose points are on a circle and whose directions make a constant angle α with the directions of the circle. A *flat field* consists of the ∞^2 elements co-circular with a fixed element. By means of a certain representation R whereby elements are represented by points of a three-space, it is found that the turbines and flat fields of the plane are pictured by the straight lines and the planes of space, respectively. In this representation, fields which correspond to the quadric surfaces of space are called *quadric fields*. It is found that *the quadric fields may be classified into nine distinct classes with respect to the whirl-motion group G_6*. Finally, the invariants for the quadric fields are found with respect to G_6. (Received January 10, 1938.)

111. J. L. Kelley: *A metric related to property S.*

It has been shown by Mazurkiewicz and by G. T. Whyburn that the metric for any connected, locally connected set may be redefined, homeomorphically, so that the neighborhood of a point is connected. As the solution to a problem proposed by G. T. Whyburn, a metric with the following properties has been derived: If the space is connected and locally connected, under the new definition of the metric, the neighborhood of a point is connected. If the space, furthermore, has property S, the ϵ neighborhood of any subset, for any ϵ, has property S. (Received January 29, 1938.)

112. D. H. Lehmer: *A factorization theorem applied to tests for primality.*

The writer has given several tests for primality, based on the converse of Fermat's theorem, which have been used during the last decade for the identification of many large primes. In order to apply any of these tests to the number N, some suitable divisor of $N-1$ must be known. For a general N, this requirement may be difficult to fulfill. However, the most interesting values of N are of the form $Q_n(a)$, where a is an integer, and $Q_n(x)$ is the irreducible cyclotomic polynomial whose roots are the primitive nth roots of unity, as, for example, the numbers of Fermat and Mersenne. This paper gives a theorem for the factorization of $Q_n(a)-1$ in terms of the factors of $Q_k(a)$ where $k<n$. There are three cases according as the number of odd prime factors of n is 0, 1, or more. (Received January 28, 1938.)

113. Howard Levi: *A characterization of the values assumed by polynomials.*

The difference quotient $f(z+h)/h-f(z)/h=k$ defines h as a function of z and k. If $f(z)$ is a polynomial of degree n, h is algebraic and its $n-1$ branches $h_1, \cdots, h_{n-1}$ satisfy $\sum_1^{n-1} h_i = -nz+\text{const.}$, $(n>2)$. It is also shown that if the difference quotient defines $n-1$ distinct functions $h_1, \cdots, h_{n-1}$, such that $\sum_1^{n-1} h_i = -nz+\text{const.}$, $(n>2)$, and if $f(z)$ is integral, then $f(z)$ is a polynomial of degree n. Both results are extended to functions of N variables. (Received January 17, 1938.)

114. A. N. Lowan: *On wave motion for infinite domains.*

The object of this paper is the integration of the non-homogeneous differential equation of wave motion $\nabla^2 u(P, t) = (a^2\partial^2/\partial t^2 + 2b\partial/\partial t)\, u(P, t) + \phi(P, t)$ where P is a point in the infinite space of one, two, or three dimensions, the solution satisfying the initial conditions: $\lim_{t\to 0} u(P, t) = f(P)$, $\lim_{t\to 0} \partial u(P, t)/\partial t = g(P)$. The paper is divided into four sections as follows: (1) one-dimensional non-resisting medium, (2) one-dimensional resisting medium, (3) two-dimensional resisting medium, (4) three-dimensional resisting medium. The method consists of reducing the given problem to one characterized by a non-homogeneous ordinary differential equation for the function $u^*(P, p)$ which is the Laplace transform of the given function $u(P, t)$. The solution of the differential equation for $u^*(P, p)$ is obtained with the aid of a certain well known theorem in conjunction with the Fourier integral identities for one, two, and three dimensions. The transition from the solution for the u^*'s to the solutions for the u's is obtained with the aid of some standard methods in the operational calculus in conjunction with certain known identities involving infinite integrals whose integrands contain one or several Bessel functions. (Received January 20, 1938.)

115. N. H. McCoy: *Concerning matrices with elements in a commutative ring.*

In this paper, most of the fundamental theorems on matrices having to do with the characteristic function and the minimum function are suitably generalized to the case of matrices with elements in an arbitrary commutative ring R with unit element. If A is a matrix with elements in R, the ideal $\mathfrak{M}$ in $R[\lambda]$ of all polynomials $h(\lambda)$ such that $h(A)=0$ is the required analog of the minimum function, and a leading result is an explicit determination of this ideal. If $f(\lambda)$ is the characteristic function of A, then it is found that the ideals $(f(\lambda))$ and $\mathfrak{M}$ have the same prime ideal divisors, this being a generalization of the familiar theorem that the distinct linear factors of the characteristic function and of the minimum function are identical. Other generalizations of a similar nature are also obtained, one of them being an analog of a known theorem concerning the characteristic roots of a polynomial in several matrices, not necessarily commutative (cf. this Bulletin, vol. 42 (1936), pp. 592–600). (Received January 12, 1938.)

116. W. T. Martin and Norbert Wiener: *Taylor's series of functions of smooth growth in the unit circle.*

The authors are concerned with functions with positive coefficients analytic in the unit circle. By studying the behavior of a function along the positive real axis, relations among the coefficients are determined. The results are the best possible of this type. (Received January 8, 1938.)

117. C. B. Morrey: *The existence of solutions of certain minimum problems for multiple integrals.*

In this paper, minimum problems for non-parametric double integrals $I(z, G)$ (of $f(x, y, z, p, q)$ over G) are discussed where (1) G is a suitably restricted region, (2) (z, p, q) may stand for $(z', \cdots, z^n, p', \cdots, p^n, q', \cdots, q^n)$, (3) $f(x, y, z, p, q) \geqq 0$ everywhere and satisfies a uniform Lipschitz condition on any bounded portion of (x, y, z)-space, and (4) $f(x, y, z, p, q)$ is convex in (p, q) for each fixed (x, y, z). The class of "potential functions of their generalized derivatives" (a concept due to G. C. Evans) is used; such functions need not be continuous. Compactness theorems are proved for certain sets of these functions. It is shown that $I(z, G)$ is lower semicontinuous with respect to the type of convergence used in the compactness theorems. If there exist numbers $m>0$ and $\alpha>1$ such that $f(x, y, z, p, q) \geqq m(|p|^\alpha + |q|^\alpha)$, it is shown that a function z of our class exists which "takes on" (in a sense described in the paper) given continuous boundary values and minimizes $I(z, G)$ among all such functions. If also $\alpha>2$ or $\alpha=2$ and $f(x, y, z, p, q) \leqq M(p^2+q^2)$, the minimizing function is continuous on the closure $\bar{G}$. Analogous results hold for n-tuple integrals. (Received January 28, 1938.)

118. Tadasi Nakayama: *A note on the elementary divisor theory in non-commutative domains.*

The reduction of a matrix from a non-commutative domain of integrity to a van der Waerden-Wedderburn normal form (*Moderne Algebra*, vol. 2, 1932; Journal für Mathematik, vol. 167 (1932)), as well as to a Jacobson normal form (Annals of Mathematics, vol. 38 (1937)), can be achieved under the mere assumption that all right and left ideals are principal; one does not need to assume the so-called euclidean division process. Moreover, the diagonal elements in a Jacobson normal form are determined by the original matrix uniquely up to similarity. (Received January 14, 1938.)

119. B. J. Pettis: *On linear functionals and completely additive set functions.*

If T is an abstract space for which the generalized measure problem (Banach and Kuratowski, Fundamenta Mathematicae, vol. 14 (1929), p. 127) has a negative answer, then a sharper form can be given to the Lebesgue theorem concerning the decomposition of functions completely additive over an additive family of measurable sets; it is also possible to give the general form of the linear functionals over these functions. The extension can be made to the case of functions having their values in a linear normed complete space. (Received January 27, 1938.)

120. J. F. Randolph: *Linear equations in an infinite number of unknowns.** Preliminary report.

Certain special cases of a system of an infinite number of linear equations in an infinite number of unknowns have been studied. The method most closely related to that of a finite set of equations in a finite number of unknowns is perhaps that of von Koch, who obtains a unique bounded solution when the double array of coefficients of the unknowns is subjected to conditions symmetrical about the main di-

* This paper is chiefly the work of the late Professor D. C. Gillespie of Cornell University.

agonal and the constants form a bounded set. In this paper the same result is obtained, but the assumptions made concerning the coefficients are not symmetrical about the main diagonal. The fact that a system in which all coefficients above the diagonal are zero and those in the diagonal are not zero has a unique solution is cited as a partial explanation of this lack of symmetry. The conditions here neither imply nor are implied by those of von Koch. (Received January 27, 1938.)

121. L. B. Robinson: *On the analytic prolongation of a singular solution of the equation of Izumi.*

The author has found a singular solution $f(\lambda, x)$ of the equation of Izumi. Except at the origin, $f(\lambda, x)$ is holomorphic with respect to x. It also converges when $|\lambda| < 1$ and is meromorphic. By means of the formula $U_{p+1}^{(n)} = U_p^{(n)} + x(U_p^{(n)})^2/n$ due to Goursat, $f(\lambda, x)$ can be approximated by a series of polynomials. (Received February 3, 1938.)

122. J. B. Rosser and R. J. Walker: *The algebraic theory of diabolic magic squares.*

A square matrix of order n is said to admit a path $\{a, b\}$ if $(a, b, n) = 1$, and if, for all i and j, $\sum_{x=1}^{n} a_{i+ax, j+bx} = N$, the subscripts being reduced modulo n. The matrix is an algebraic diabolic square (a.d.s.) if it admits $\{0, 1\}$, $\{1, 0\}$, $\{1, 1\}$, and $\{1, n-1\}$; and an a.d.s. is a numerical diabolic square (n.d.s.) if the elements consist of the integers $1, 2, \cdots, n^2$ (so that $N = n(n^2+1)/2$). There is no n.d.s. of order 3, or of any order $\equiv 2 \pmod 4$. It is shown that there are n.d.s. of all other orders, and that one can construct at least $(n!)^2\,\theta(n)$ n.d.s. of order n by the classic "step-process," where $\theta(2^\alpha) = 0$, $\theta(p^\alpha) = p^{2\alpha-2}(p-3)(p-4)$ if p is an odd prime, and $\theta(mn) = \theta(m)\theta(n)$ if $(m, n) = 1$. If $n \not\equiv 2 \pmod 4$ and $\neq 3$ or 5, there exist n.d.s. of order n not constructible by the step-process. Hyper-a.d.s., which admit additional paths, are considered, and general (algebraic) solutions obtained. Using these, one can prove that a n.d.s. which admits all but two paths must be constructible by the step-process. A consequence of this is that all n.d.s. of order 5 are constructible by the step-process, and there are exactly 28,800 of them. Some of these results are extended to cubes. (Received January 28, 1938.)

123. O. K. Sagen: *A quasi-canonical form in the ring of integers.*

A non-singular integral quadratic form in n variables is reduced by a unimodular transformation to a unique form with the same leading coefficient. The reduction is a modification of that for the rational canonical form. Its interest lies in its application to the study of integer representations by quadratic forms in more than two variables. General results for a genus of ternary forms are readily obtained. (Received January 28, 1938.)

124. Wladimir Seidel and J. L. Walsh: *On the derivatives of functions analytic in the unit circle.*

Let $w = f(z)$ map conformally $|z| < 1$ onto a Riemann domain S. Let $R(w)$ denote the radius of the largest smooth circle on S with center w. If either $f(z)$ omits two values or $R(w)$ is bounded independently of w, then a necessary and sufficient condition for (1) $|f'(z_n)|\,(1-|z_n^2|) \to 0$, with $w_n = f(z_n)$ bounded, is $R(w_n) \to 0$. This condition is necessary but not sufficient for the general class of functions analytic for $|z| < 1$. The approach in (1) may be arbitrarily slow, even when $f(z)$ is bounded, univalent, and continuous for $|z| \leqq 1$. If $f(z)$ is univalent, one has $|f^{(k)}(z)|\,(1-|z|^2)^k$

$\leqq P_{k-1}(|z|)\cdot R[f(z)]$, where $P_{k-1}(|z|)$ is a polynomial in $|z|$ of degree $k-1$ with positive coefficients. For $k=1$ this inequality is due to Littlewood and Macintyre. If $f(z)$ is bounded, similar results can be obtained for $k=1$. If $f(z)$ is univalent, then at almost all points of the circumference $|f'(z)|(1-|z|)^{1/2}\to 0$, as z approaches the circumference angularly. (Received January 21, 1938.)

125. J. J. Stoker: *An eigenvalue problem in elasticity with continuous spectrum.*

A plane plate subjected to compressive or shear forces in the plane of the plate will buckle when such loads attain certain critical values, the determination of which leads to an eigenvalue problem involving a partial differential equation of fourth order. Two special cases have been found which lead to eigenvalue problems with continuous spectra—a phenomenon hitherto encountered in only one other physical problem (with a bounded domain) known to the author, that is, in the case of the Schrödinger equation in quantum mechanics. (Received January 27, 1938.)

126. J. J. Stoker: *On unbounded convex point sets.*

Let S^3 be an unbounded convex point set in three-dimensional euclidean space, $\overline{S}^3$ the set of its boundary points. It is shown that $\overline{S}^3$ is topologically one of the following forms: (1) the plane, (2) a pair of distinct planes, (3) an open cylinder. The spherical image of S^3 is shown to lie on a hemisphere; conditions under which the latter set is open or closed are given. (Received January 27, 1938.)

127. W. C. Strodt: *Sequences of systems of algebraic differential equations.*

Let $\Sigma_1, \Sigma_2, \cdots$ be a sequence of closed irreducible systems in the unknowns $y_1, \cdots, y_n$, such that the manifold of Σ_i is a proper part of the manifold of Σ_{i+1}, $(i=1, 2, \cdots)$. Let Σ be the set of forms common to the Σ_i. Then Σ is a closed irreducible system, and a necessary and sufficient condition for an ordered set $y_1(x), \cdots, y_n(x)$ of n functions analytic in a region $\mathfrak{B}$ to be a solution of Σ is that there exist a point x_0 in $\mathfrak{B}$ such that for every positive integer m and every positive number ϵ there is a positive integer i such that Σ_i has a solution $\bar{y}_1(x), \cdots, \bar{y}_n(x)$ analytic at x_0 with $|\bar{y}_{ij}(x_0)-y_{ij}(x_0)|<\epsilon$, $(i=1, \cdots, n; j=0, 1, \cdots, m)$, where the second subscript is an index of differentiation. When $y_1(x), \cdots, y_n(x)$ is a solution of Σ, the set of points x_0 consists of all the points in $\mathfrak{B}$ except for a set which is at most denumerable. (Received January 10, 1938.)

128. Otto Szász: *On the jump of a function determined by its Fourier series.*

Let $f(x)$ be integrable and of period 2π; hence it has a Fourier series. Denote by $\sigma_n(x)$ the arithmetic means of the partial sums of the series. It is well known that $\sigma_n(x)\to\{f(x+0)+f(x-0)\}/2$; to determine the jump $f(x+0)-f(x-0)=D(x)$, several devices have been used. The author gives a new and effective way for the solution of this problem. Denote by $\bar{\sigma}_n(x)$ the trigonometric polynomial conjugate to $\sigma_n(x)$; the result is $D(x)=\lim_{n\to\infty}(\pi/\log 2)\{\bar{\sigma}_{2n}(x)-\bar{\sigma}_n(x)\}$, whenever $D(x)$ exists in the usual sense or as an integral mean. (Received January 27, 1938.)

129. A. E. Taylor (National Research Fellow): *Linear differential systems and linear operations analytic in a parameter.*

For each complex t in the circle $|t|<r$ let A_t be a bounded linear transformation

in the complex Banach space E, and let $A_t x$ be analytic (differentiable) for each $x \epsilon E$. The author considers the differential systems $dx/dt = A_t x$, $x(0) = y$ and $dx/dt = A_t x + z(t)$, $x(0) = 0$, where y is arbitrary in E and $z(t)$ is analytic, $|t| < r$, with values in E. The unique analytic solution of the first system is given by $x = B_t y$, where B_t is a linear operation analytic in t (as A_t is). For each t, $|t| < r$, B_t defines a one-to-one mapping of E on itself, with analytic inverse B_t^{-1}. A solution of the non-homogeneous system is then $x = B_t \int_0^t B_s^{-1} z(s) ds$. In developing the theory use is made of an "adjoint" system involving the conjugate operation A_t^* defined in the linear functional space conjugate to E. The results include many of the known theorems on differential systems in an infinite number of unknowns, as well as the classical theory. (Received January 24, 1938.)

130. A. E. Taylor (National Research Fellow): *Linear operations which depend analytically on a parameter.*

Let E, E' denote complex Banach spaces. Let A_t be a bounded linear operation with domain E and range in E' for each t in a region Δ of the complex plane. If for each $x \epsilon E$, $A_t x$ is differentiable as a function on Δ to E', it is said that A_t depends analytically on t. It is then also analytic (differentiable) as a function on Δ to the space of bounded linear operations on E to E' when this space is normed by the bound of the operation. Application of this result to linear functionals yields results of the following sort: In order that a sequence $\{x_n(t)\}$ of numerical functions define an analytic function on Δ to (l_p), $(p \geqq 1)$, it is necessary and sufficient that (i) $x_n(t)$ be analytic in Δ, (ii) $\sum_1^\infty |x_n(t)|^p < \infty$ in Δ, and the series be bounded in each bounded closed subregion of Δ. Corresponding theorems are obtained for the spaces (c), (m), L_p, $(p \geqq 1)$. The set of points in Δ for which A_t admits a bounded inverse with domain E' is either null or open, and in the latter case the inverse is analytic also. (Received January 24, 1938.)

131. S. M. Ulam: *On bounded transformations of spaces.* Preliminary report.

A continuous transformation T of a metric space into itself will be said to have the bound k, if k is the g.l.b. of numbers l such that $\rho(T^n(p), p) \leqq l$ for all p and n. A class of locally connected continua C is determined, such that there exist positive numbers $k(C)$ with the property that any T transforming C into itself and having a bound $< k(C)$ must have a fixed point $T(p_0) = p_0$. (Received January 27, 1938.)

132. S. M. Ulam: *Set-theoretical invariants of the product operation.*

Let K be a class consisting of countably many subsets A_n of a set E (for example, rational intervals on the line). Let $B(K)$ be the smallest class of sets containing K and closed with respect to the operations of set-difference and countable addition. Properties of classes which can be imbedded in a $B(K)$, in particular Borel classifications, are established. Let A be an abstract set and L the class of all subsets of $A^2 = A \times A$ of the form $A_1 \times A_2$, $A_1, A_2 \subset A$. Properties of a Borel classification of $B(L)$ are determined, in particular invariance with respect to product-isomorphisms (transformations of the form $(a_1, a_2) \to (f(a_1), f(a_2))$, where f is an arbitrary one-to-one transformation of A into itself). (Received January 27, 1938.)

133. Morgan Ward: *The law of apparition of primes in a Lucasian sequence.*

A sequence $u_0, u_1, u_2, \cdots$ of rational integers is said to be Lucasian if it satisfies

a linear recursion relation with constant integral coefficients and if u_n divides u_m whenever n divides m. The author considers Lucasian sequences of a form sufficiently general to include all heretofore published instances and reduces the problem of determining à priori all terms of such a sequence divisible by a preassigned prime modulus to the fundamental problem of determining the period of a mark in a finite field. This paper will appear shortly in the Transactions of this Society. (Received January 13, 1938.)

134. Hassler Whitney: *Tensor products of abelian groups.*

Given two abelian groups G and H, let S be the set of all finite "sums" $g_1 \cdot h_1 + \cdots + g_n \cdot h_n$ (any n). If any two elements of S which may be proved equal with the help of the two distributive laws are identified, one obtains an abelian group, the *tensor product* of G and H. Fundamental properties are studied, including relations with operator rings. If G and H are linear, it is assumed also that $rg \cdot h = g \cdot rh$ (real r). If the groups are topological and separable, a topology may be introduced into the product. The elements of tensor analysis, including contraction and covariant differentiation, are simply expressed with the help of tensor products (without using coordinate systems). (Received January 24, 1938.)

135. Leonidas Alaoglu: *Weak convergence of linear functionals.*

The definition of weak convergence of sequences of linear functionals given by Banach (*Opérations Linéaires*, p. 122, §4) is extended by the use of the Moore-Smith limit. A directed set $(f_\alpha | \alpha)$ of linear functionals on a space X of type B is defined to be weakly convergent to the functional f if the set is bounded, and if $\lim_\alpha f_\alpha(x) = f(x)$ for every x in X. The topology so defined in the adjoint space X^* is equivalent to a Hausdorff topology in the bounded subsets of X^*, and is such that weakly closed and bounded subsets are bicompact. By means of this result it is shown that a linear subspace of X^* is regularly closed if and only if it is weakly closed. The other theorems of Banach (chap. 8) which refer to weak convergence are similarly extended, the condition that X be separable being removed. (Received February 1, 1938.)

136. A. A. Albert: *On cyclic algebras.*

Let Z of order n over K be a direct sum of t equivalent cyclic fields over K, so that Z has an automorphism S of order n over K. Define a cyclic system (Z, S) of all pairs Z', S' with Z' equivalent to Z under a correspondence such that S' goes into S. Such systems were considered by O. Teichmiller, Deutsche Mathematik, vol. 1 (1936), pp. 197–238. He connected them with corresponding cyclic algebras (Z_L, S, x) over $L = K(x)$, and used the known results on cyclic algebras to prove a number of corresponding theorems on the cyclic systems. The author proves the theorems on the cyclic systems by elementary commutative arguments and uses the results to provide new proofs of the theorems on cyclic algebras. In particular let Z be a cyclic field, d be a divisor of n, Y be the subfield of Z of degree nd^{-1}. Then the present work provides the first simple proof of the theorem $(Z, S, \gamma)^d \sim (Y, S, \gamma)$. (Received March 1, 1938.)

137. C. R. Cassity: *The maps determined by the principal curves associated with five and six points in the plane.*

The group associated with five points is of order 1920. The map has twenty four-sided and sixteen five-sided regions. It has only partial symmetry, since only one hundred and sixty elements can be interpreted as topological transformations of the map into itself. A study of the map obtained on the cubic surface by the cubic curves on

six points of the plane is the principal purpose of the present paper. The case which leads to twenty-seven real lines with no three concurrent is that under consideration. The number and types of regions were given by Klein (Mathematische Annalen, vol. 6 (1873), p. 570) and Zeuthen (Mathematische Annalen, vol. 8 (1875), pp. 1–30) The author shows that there is only one type of map and that it has no symmetry whatever. The surface can be put onto itself in only the identical way. The statement of Zeuthen that any of the fifteen lines, each of which forms the boundary of two triangles, can be "shifted across" either triangle independently of the other is examined and a method is derived for determining the number of other "shifts" which must precede any desired one. The methods are largely synthetic. (Received February 23, 1938.)

138. Harold Chatland: *A note on the asymptotic Waring problem for homogeneous polynomial summands.*

In a paper appearing in the January, 1938, issue of the Annals of Mathematics, the author proved that the number of homogeneous polynomials of the form $\sum_{r=0}^{s} a_r x^{n-r} y^r$ which suffices to express integers greater than a certain (large) integer is about two-thirds of the number of nth powers required. By a new treatment of some inequalities appearing in the paper mentioned, the number of homogeneous polynomial summands required to express such integers has been reduced to about one-half the number of nth-power summands. The restriction $a_0=a_1=a_2=1$ on the integral coefficients is retained in this paper. (Received February 24, 1938.)

139. J. M. Dobbie: *A generalized Lambert series.*

Let $h(x)$ be a function which is analytic interior to $|x|=1$ and which has the value 1 at $x=0$. Then the series $\sum_1^\infty b_n x^{\lambda n} h(x^n)$, ($\lambda$ a positive integer), is called a generalized Lambert series and is discussed as to its convergence properties, necessary and sufficient conditions for expanding a function in such a series, and the existence of a natural boundary for the sum function. On this latter subject are proved theorems analogous to the theorems for ordinary Lambert series proved by Knopp (Journal für die reine und angewandte Mathematik, vol. 142 (1913), pp. 283–315) and by Hardy (Proceedings of the London Mathematical Society, (2), vol. 13 (1913), pp. 192–198). The hypotheses on the coefficients b_n are stated in terms of Cesàro summability for a non-negative integer, while the approach is radial. (Received March 1, 1938.)

140. Nelson Dunford: *One parameter groups of linear transformations.*

A one parameter group T_s, $(-\infty<s<\infty)$, of linear transformations on a Banach space X is continuous in the strong topology of the ring of operators on X if: (i) $\|T_s\|$ is bounded in some interval about $s=0$; (ii) for each x, the set $T_s x$, $(-\infty<s<\infty)$, is separable; and (iii) for each y in $\overline{X}$ and x in X, the function $yT_s x$ is measurable in the sense of Lebesgue. (Received February 26, 1938.)

141. Arnold Emch: *New properties of the cubic surface.*

In 1931 the author found a new normal form of the cubic surface which was published in the American Journal of Mathematics, vol. 53, pp. 902–910. This form permits one to prove algebraically many properties of the cubic which up to the present time had been established only by synthetic methods. Moreover a new connection is shown between the Δ_{18}-configuration of syzygetic pencils and the Steiner-Dixon

summit-planes of associated triples of couples of conjugate Steinerian trihedrals. It is shown that a general cubic surface can be put into this normal form in at least forty different ways; in one way at least for each of the forty summit-planes. (Received February 18, 1938.)

142. Aline H. Frink and Orrin Frink: *Polygonal variations.*

Arcs minimizing an integral $\int f(x, y, y')dx$ may exist even though many of the partial derivatives of $f(x, y, y')$, which occur in the usual treatment of the calculus of variations, do not exist. Hence it is important to have proofs of the familiar necessary conditions which require the existence of as few partial derivatives of f as possible. It is shown in this paper that, by giving the dependent variable variations whose graphs are polygonal lines of proper shape depending on a parameter ϵ, and by assuming the existence of only the single partial derivative $f_{y'}$, it is possible to derive the Weierstrass necessary condition and a generalization of the Euler equation. The ordinary form of the Euler equation can be derived by this method without the use of integration by parts or Du Bois Reymond's lemma. A slight generalization of the Legendre necessary condition is also obtained, assuming the existence of only the single generalized second partial derivative $f_{y'y'}$. The method extends to the case of more than one dependent variable, and to the parametric problem. (Received February 24, 1938.)

143. Einar Hille: *On semi-groups of transformations in Hilbert space.*

If $T_\alpha x$ is a family of self-adjoint, positive definite transformations in Hilbert space, defined for $\alpha>0$ and having the semi-group property $T_\alpha[T_\beta]=T_{\alpha+\beta}$, and if $\|T_\alpha x\| \leqq \|x\|$, then $(T_\alpha x, y)=\int_0^\infty e^{-\alpha\lambda}d(E(\lambda)x, y)$. Here $E(\lambda)$ is the resolution of the identity of a self-adjoint, positive definite transformation $Ax=(-\log T_1)x$. In particular, $(T_\alpha x, y)$ is an analytic function of α, holomorphic and bounded for $\Re(\alpha)>0$. The author has determined the transformation A for the transformations associated with the Gauss-Weierstrass and the Poisson singular integrals. (Received February 26, 1938.)

144. Einar Hille: *On the absolute convergence of polynomial series.*

If the terms of a series are polynomials with real zeros, the absolute convergence of the series at a non-real point implies absolute convergence in a certain region. Using elementary methods, the author determines this region under various assumptions on the distribution of the zeros. (Received February 26, 1938.)

145. Charles Hopkins: *The half-group of cosets belonging to a group.*

Let G denote any group; let Ω denote a set of operators for G which contains at least all the inner automorphisms of G; and let S denote the set of all admissible subgroups H of G. Each H gives rise to the quotient group $\Gamma=G/H$, and each Γ is isomorphic with the group of cosets of G with respect to H. Under the usual definition of multiplication for complexes the set of all distinct cosets of G with respect to all elements of S will form a half-group $\Gamma(G)$. Certain properties of $\Gamma(G)$ are discussed. For the case when S contains a finite number n of elements it is shown that the "extended group-ring of G," that is, the hypercomplex system $R(G)$ (over a field F) which has as a basis the elements of $\Gamma(G)$, is the direct sum of n invariant subrings $\overline{R}_1+\cdots+\overline{R}_n$, where each $\overline{R}_i$ is ring-isomorphic with the regular group-ring of Γ_i over F. (Received March 1, 1938.)

146. F. B. Jones: *Concerning R. L. Moore's Axiom* 5.

Consider the following axiom: Axiom 5_0. *If A is a point of a region R and B is a point distinct from A, there exists in R a compact continuum separating A from B.* If a space satisfies Axioms 0–4 of R. L. Moore's *Foundations of Point Set Theory* and Axiom 5_0, it will also satisfy Moore's Axiom 5. (Received February 25, 1938.)

147. H. L. Krall: *On certain differential equations for Tchebycheff polynomials.*

The purpose of this paper is to find conditions on the coefficients $\{l_{ij}\}$ in order that the differential equation $\sum_{i=0}^{r}(\sum_{j=0}^{i} l_{ij}x^j)y_n^{(i)}(x)=\lambda_n y_n(x)$, ($\{l_{ij}\}$ constants, λ_n a parameter), will have a set of orthogonal polynomials as solutions. Conditions are found which are necessary and sufficient for the existence of such a differential equation. The classical polynomials of Jacobi, Hermite, and Laguerre give us examples of polynomials which satisfy such equations. An example of a non-classical set of orthogonal polynomials is given which also satisfies an equation of the above type. (Received February 26, 1938.)

148. O. E. Lancaster: *Sequences defined by non-linear algebraic difference equations.*

This paper treats of sequences defined by difference equations of the form $\Phi(s_{n+m}, s_{n+m-1}, \cdots, s_n, n)=0$, where Φ is a polynomial with real rational coefficients in its arguments $s_{n+m}, s_{n+m-1}, \cdots, s_n, n$, and n is an integer. The considerations are confined to those sequences which approach finite limits. Some criteria, sufficient to insure that the sequences defined by a given difference equation approach a constant limit α, are developed. A study is made of the relation between the rate of convergence of the sequences and the order of the difference equation, and between the rate of convergence of the sequences and the degree of n in the coefficients of the difference equation. Special attention is given to first order difference equations whose rational sequences approach $\beta^{1/p}$, where p is an integer, β is a rational number, and $\beta^{j/p}$, ($j=1, 2, \cdots, p-1$), is an irrational number. (Received February 23, 1938.)

149. Saunders MacLane: *Transcendence degrees and p-bases in terms of non-modular lattices.*

R. Baer has remarked that the invariance of the transcendence degree of a field and of the rank of certain abelian groups should have a common lattice-theoretic origin. This paper analyzes the type of lattice concerned: a continuous lattice satisfying a point-exchange axiom, a finiteness axiom, and an axiom insuring the existence of sufficient points. In such a lattice a "basis" of independent points can always be constructed. Its cardinal number is an invariant. This result includes the two invariants above and yields a new theorem for the p-basis of an inseparable algebraic extension of a field of characteristic p. The crucial point-exchange axiom in the lattice is weaker than the usual modular law. For other applications, this exchange axiom can be stated in two alternative forms, which, like the modular law, involve no reference to points: (I), $c>d=(a+b)\cdot c$ and $ab\geqq d$ imply the existence of a c_1 with $d<c_1\leqq c$ and $ab=(a+c_1)b$; (II), $b\leqq a+c$, b not $\leqq c$ imply the existence of c_1 and c_2 with $c_1+c_2\leqq c$, $0<c_1\leqq a+b+c_2$, and b not $\leqq a+c_2$. (Received February 11, 1938.)

150. P. T. Maker: *Conditions on $u(x, y)$ and $v(x, y)$ necessary and sufficient for the regularity of $u+iv$.*

This paper extends the Looman-Menchoff theorem as follows: Let $\mathfrak{F}$ be a class of continuous functions defined in the open set G in the complex plane, and let K be a class of G for which regularity of any function of $\mathfrak{F}$ in $G-E$, $(E\varepsilon K)$, implies its regularity in G. If $f(z), [=u(x, y)+iv(x, y)]$, $\varepsilon\mathfrak{F}$, and the Dini partial derivates of u and v are infinite on $\sum_{n=1}^{\infty}E_n$, $(E_n\varepsilon K$, and closed in $G)$ at most, with the Cauchy-Riemann equations holding almost everywhere where the partials exist, $f(z)$ is regular in G. When u and v are of bounded variation (Tonelli), it is shown that if almost all parallels to the axes intersect E in a denumerable set at most, then $E\varepsilon K$. For u and v absolutely continuous, $f(z)$ is shown to be regular if the Cauchy-Riemann condition stated above holds. It follows that $f(x, y)$ is harmonic if and only if $f(x, y)$ is absolutely continuous and $\int(df/dn)ds=0$ around "almost all" rectangles in G. (Received February 28, 1938.)

151. J. D. Mancill: *On volumes bounded by cylindrical surfaces.*

The problem with which this note is concerned is that of determining certain volumes common to two given cylindrical surfaces. The usual procedure of integral calculus leads to extremely difficult integration even in the simplest problems unless the cylindrical surfaces have their elements perpendicular. It is shown in this paper how these procedures may be slightly altered to simplify greatly the integration in the general case. Two very interesting properties of cylindrical surfaces are also derived. This note is prompted by repeated requests for solutions of problems of this nature. (Received February 12, 1938.)

152. F. D. Murnaghan: *The generalized Clebsch-Gordan series.*

The Clebsch-Gordan series furnishes the analysis of the Kronecker product (a) of irreducible representations of the two-dimensional unimodular linear group and (b) of irreducible representations of the three-dimensional rotation group. It is shown here that (a) is a special instance of the formula: $D(\lambda)\times(D(\mu)=D(\lambda, \mu)+D(\lambda+1, \mu-1)+\cdots+D(\lambda-\mu)$ which is valid for the n-dimensional full linear group; and that (b) is a special instance of the formula: $D(\lambda)-\times D(\mu)=\{D(\lambda_1+\lambda_2)+D(\lambda_1+\lambda_2-2)+\cdots+D(\lambda_1-\lambda_2)\}+\{D(\lambda_1+\lambda_2-1,1)+D(\lambda_1+\lambda_2-3,1)+\cdots+D(\lambda_1-\lambda_2+1,1)\}+\{D(\lambda_1+\lambda_2-2,\ 2)+D(\lambda_1+\lambda_2-4,\ 2)+\cdots+D(\lambda_1-\lambda_2+2,\ 2)\}+\cdots+D(\lambda_1,\ \lambda_2)$; for example, $D(3)\times D(2)=D(5)+D(3)+D(1)+D(4,\ 1)+D(2,\ 1)+D(3,\ 2)$ which is valid for the full n-dimensional real orthogonal group. When $n=3$ or 2, the $D(\mu_1, \mu_2)$ which depend on two labels must be modified according to the following "modification" rules. When $n=2$: all $D(\mu_1, \mu_2)$ for which $\mu_2>0$ are dropped; all $D(\mu_1, 2)$ are replaced by $-D(\mu_1)$; all $D(\mu_1, 1)$ for which $\mu_1>1$ are dropped; $D(1, 1)$ is replaced by $D^*(0)$, the star denoting the associated representation. When $n=3$: all $D(\mu_1, \mu_2)$ for which $\mu_2>1$ are dropped; all $D(\mu_1, 1)$ are replaced by $D^*(\mu_1)$. For the rotation subgroup of the full real orthogonal group, the stars may be dropped. (Received February 24, 1938.)

153. F. D. Murnaghan: *The analysis of the representations of the real n-dimensional orthogonal group induced by irreducible, rational, integral representations of the full linear group.*

Denoting by k the integer $n/2$ or $(n-1)/2$ according as n is even or odd, one can

give the analysis of the representation $D(\lambda_1, \cdots, \lambda_j)$, $(j=1, 2, \cdots, k)$, by the formula $D(\lambda_1, \cdots, \lambda_j) = [\prod_{p,q}(1-\xi_p\xi_q)^{-1}] \cdot \Delta(\lambda_1, \cdots, \lambda_j)$, in which p and q vary from 1 to j, $p \leqq q$, and $j=1, \cdots, k$. In the product, ξ_p is an operator which reduces the label attached to λ_p by unity, and ξ_q is a similar operator. The Δ are irreducible representations of the orthogonal group; for example, $D(\lambda_1) = \Delta(\lambda_1) + \Delta(\lambda_1-2) + \Delta(\lambda_1-4) + \cdots$; $D(4, 2) = \Delta(4, 2) + \Delta(4) + \Delta(3, 1) + \Delta(2^2) + 2\Delta(2) + \Delta(0)$. The above formula is also valid when $j>k$, but in such cases the $\Delta(\lambda_1, \cdots, \lambda_p)$, $p>k$, must be "modified" in accordance with "modification rules" given in the preceding abstract for $j=k+1$. For example, $n=2$, $D(4, 2) = \Delta(2) + \Delta(0)$; $n=3$, $D(4, 2) = \Delta(4) + \Delta^*(3) + 2\Delta(2) + \Delta(0)$, the star denoting the associated representation. The method here described is available (and simpler in application) for the complex group. (Received February 24, 1938.)

154. W. T. Puckett: *Concerning local connectedness under the inverse of certain continuous transformations.*

A continuous transformation $T(A) = B$ on the compact set A is said to have property β relative to (A, B) provided that for $\epsilon>0$ there exists a $\delta>0$ such that every pair of points, $p, q \in T^{-1}(b)$, $(b \in B)$, with $\rho(p, q) < \delta$ lie in a connected subset of A of diameter $\leqq \epsilon$. In case A is locally connected, K is any closed subset of B with $F(B-K)$ finite, and $T^{-1}(b)$, $(b \in B)$, is locally connected, then T has property β relative to $[T^{-1}(K), K]$. Suppose T interior and B locally connected. Then in order that A be locally connected it is necessary and sufficient that T have property β relative to (A, B). In case A is a continuum and $T(A) = B$ is 0-regular (see A. D. Wallace, abstract 44-3-161), T has property β relative to (A, B). Thus local connectedness is invariant under the inverse, since T is also interior. If T is continuous and the inverse of every point is finite (countable), then the property of being a regular (rational) curve is invariant under the inverse. In case T is interior and finite, local connectedness is invariant under the inverse. (Received February 26, 1938.)

155. W. T. Puckett: *Concerning the 1-dimensional Betti group under interior transformations.*

Let $T(A) = B$ be an interior transformation on the compact set A. Let N be a subcontinuum of A such that $N \cdot \overline{(A-N)} = Q$ is connected and $p^1[T(Q)] = 0$. Then for $\epsilon>0$ there exists a $\delta>0$ such that if Z is any 1-dimensional rational δ-cycle in $T(N)$, there exists a 1-dimensional ϵ-cycle C in N such that $f(C) \sim_\epsilon Z$. Consequently, if V is any rational Vietoris (or true) cycle in $T(N)$, there exists a Vietoris (or true) cycle W in N such that $f(W) \sim_\epsilon V$. This is a strengthening of a theorem by G. T. Whyburn (in a paper to appear in an early number of the Duke Mathematical Journal). From this paper it follows that if $p^1[T(N)]$ is finite the group $B^1_R(N)$ maps homomorphically onto $B^1_R[T(N)]$, and $p^1[T(N)] = p^1(N)$. (Received February 26, 1938.)

156. M. S. Robertson: *Piecemeal univalency of analytic functions.*

The properties of functions $f(z)$, $f'(0)=1$, which are regular and univalent within the unit circle have been studied in detail by many authors. In this paper, a more general class of functions $f(z) = z^k + \sum_{k+1}^{\infty} a_n z^n$, regular in the unit circle, is considered. These functions are not necessarily univalent within the unit circle as a whole, but only in pieces of the circle consisting of all the sectors of the circle having an aperture $\alpha\pi$, where $0 < \alpha \leqq 2$. For this class of functions, which contains as a subclass the univalent functions mentioned above, it is shown that $|f(re^{i\theta})| \leqq A(\alpha) r^k (1-r)^{-2}$,

$(1/(2\pi))\int_0^{2\pi}|f(re^{i\theta})|\,d\theta<[2/\alpha]A(\alpha)r^k(1-r)^{-1}$, $|a_n|<[2/\alpha]A(\alpha)e^n$; wherein $A(\alpha)$ is a constant depending upon α, but not upon $f(z)$. (Received February 23, 1938.)

157. W. E. Sewell: *Degree of approximation by polynomials in z and* $1/z$.

The theorem of Marcel Riesz (Jahresbericht der Deutscher Mathematiker-Vereinigung, vol. 23 (1914), pp. 354–368) on the modulus of the derivative of a polynomial in z and $1/z$ on the unit circle is extended to generalized derivatives on Jordan curves. These results are applied to the approximation of a function defined on a Jordan curve. (Received February 17, 1938.)

158. W. E. Sewell: *Note on the relation between Lipschitz conditions and degree of polynomial approximation.*

Let E, with boundary C, be a closed limited convex set and let $f(z)$ be defined on C. For each n, $(n=1, 2, \cdots)$, let there exist a polynomial $P_n(z)$ of degree n in z such that $|f(z)-P_n(z)|\leqq M/n^\alpha$, $\alpha>0$, z on C, where M is a constant independent of n and z. Then $f(z)$ satisfies on C a Lipschitz condition of order $\alpha/(t+\alpha)$, where t is a constant depending only on C. (Received February 17, 1938.)

159. R. M. Thrall: *Apolarity of trilinear forms and pencils of bilinear forms.*

In this note the idea of apolar trilinear forms, introduced earlier by the author (*Metabelian groups and trilinear forms*, American Journal of Mathematics, vol. 60 (1938)), is generalized and the main theorems are given new proofs independent of the group theoretic methods and notation used then. The importance of the apolarity concept in trilinear form classification is illustrated by a numerical application. In §3 there is given a new method of classifying singular pencils of bilinear forms, based on Dickson's minimal numbers (*Singular case of pairs of bilinear, quadratic or hermitian forms*, Transactions of this Society, vol. 29 (1927), pp. 239–253). These numbers are given by specific formulas in terms of ranks of constant matrices. (Received February 23, 1938.)

160. A. D. Wallace: *On local homeomorphisms.*

If $T(A)=B$ is a local homeomorphism (S. Eilenberg, Fundamenta Mathematicae, vol. 24 (1935), p. 35) on the locally connected continuum A, the following results are established: (1) If H is a simple closed curve in B, every component of the inverse of H is a simple closed curve, and T is topologically equivalent to $w=z^k$ on each of them. (2) If D is a dendrite in B, each component of the inverse of D is a dendrite and T is topological on each of them. (3) If H is an A-set in A, $T(H)$ is an A-set in B. (4) If x is a cut point in A, $T(x)$ is a cut point in B. (5) If A is a linear graph and X is the Euler characteristic, $X(A)=kX(B)$, where k is the multiplicity of T. (Received February 26, 1938.)

161. A. D. Wallace: *On 0-regular transformations.*

If A is a metric space and $T(A)=B$ is continuous, then T is said to be *0-regular* provided the convergence of the sequence of points x_n to the point x in B implies the 0-regular convergence of the sets $T^{-1}(x_n)$ to $T^{-1}(x)$ in A. (For the definition of 0-regular convergence see G. T. Whyburn, Fundamenta Mathematicae, vol. 25 (1935), p. 412.) Under the assumption that A is a compact continuum, the following theo-

rems are proved: (1) For each x in B, $T^{-1}(x)$ is locally connected. (2) $T=T_2T_1$, $T_1(A)=A'$, $T_2(A')=B$, where T_1 is monotone and 0-regular and T_2 is finite, of constant multiplicity, and 0-regular. (3) If x is a cut-point of A then $T(x)$ is a local separating point of B. (4) T is 0-regular on open sets in A and on each component of the inverse of a connected set. (Received February 26, 1938.)

162. A. D. Wallace: *On regular transformations.*

The continuous transformation $T(M)=N$, on the compact metric space M, is said to have property Z (Wilson, American Journal of Mathematics, vol. 54 (1932), p. 377) if for each x in N and each positive ϵ there is a positive δ and a continuum K in M containing the δ-neighborhood of $T^{-1}(x)$ and contained in the ϵ-neighborhood of $T^{-1}(x)$; T is monotone if the inverse of every point is connected. T is said to be *regular* (Wilson, loc. cit.) if it is monotone and has property Z. In this paper it is shown that a necessary and sufficient condition that T be regular is that it have property Z and that N be locally connected. (Received February 26, 1938.)

163. F. P. Welch: *Non-linear singular differential equations containing a parameter.*

This work is an investigation of the properties of solutions of the equation $y'(x, \lambda)=\lambda^p a(x, \lambda, y)$, p an integer >0, (λ a complex parameter, x on the interval (a, b)). Here $a(x, \lambda, y)$ is analytic in λ and y and indefinitely differentiable in x at the point $\lambda=\infty$, $y=0$, x on (a, b). The solutions have been studied in the neighborhood of the point $x=\lambda$. The results of W. J. Trjitzinsky's paper in Acta Mathematica, vol. 66, were used for working out integrations involved in this paper. (Received March 1, 1938.)

164. G. T. Whyburn: *Interior surface transformations.*

Let $T(A)=B$ be a light interior transformation where A (hence also B) is a compact 2-dimensional manifold. Let α and β be the boundaries (if any) of A and B respectively. Let X be the set of all points of $T^{-1}(\beta)$ of order $\neq 2$, let Y be $T(X)$ plus all points of $B-\beta$ whose inverse contains points at which T is not locally topological, and let k be the multiplicity of T on the set $A-T^{-1}(Y)-T^{-1}(\beta)$. Let r and n be the numbers of points in Y and $T^{-1}(Y)$ respectively, and let m be the number of components of $T^{-1}(\beta)-\alpha-T^{-1}(Y)\cdot T^{-1}(\beta)$ which are not simple closed curves. Then if χ denotes the Euler characteristic, it is shown that $k\chi(B)-\chi(A)=kr-n-m$. Among the many consequences of this result the following are mentioned: (1) if $m=0$ (as, for example, when $\alpha=T^{-1}(\beta)$) and $\chi(A)=\chi(B)$, then either T is topological or else $r\geqq\chi(A)$. (2) If $0>\chi(A)=\chi(B)$ and $m=0$, the transformation is necessarily topological. (3) If $rk=n$ and $m=0$ (for example, if T is locally topological), then (i) $\chi(A)$ and $\chi(B)$ vanish or fail to vanish together and (ii) if $\chi(A)=\chi(B)\neq 0$, T is necessarily topological. (Received February 28, 1938.)

165. John Williamson: *Matrices normal with respect to an hermitian matrix.*

A square matrix A, with elements in the complex number field, is defined to be normal with respect to a non-singular hermitian matrix H if $AH=Hf(A^*)$, where $f(A^*)$ is a polynomial in the conjugate transpose of A. The theory of matrices normal with respect to H is similar to that of matrices normal with respect to any matrix S conjunctively equivalent to H. When S is suitably chosen, canonical forms for mat-

rices normal with respect to S, under similar transformations by matrices which are conjunctive automorphs of S, are determined. By specializing $f(A^*)$ to be equal to A^*, the known results on the theory of the conjunctive equivalence of two non-singular pencils of hermitian matrices are obtained. (Received February 25, 1938.)

166. H. S. Zuckerman (National Research Fellow): *On the coefficients of certain modular forms belonging to subgroups of the modular group.*

A modular function $F(\tau)$ belonging to the full modular group has a single Fourier expansion about the parabolic point $\tau = i\infty$. In the case of modular functions belonging to a subgroup, however, it is necessary to consider expansions about a set of non-equivalent parabolic points. The coefficients of these expansions are obtained by a method (developed by H. A. Rademacher and H. S. Zuckerman in a recent paper) which is a modification of the Hardy-Ramanujan method. Applications to the reciprocals of the theta-functions are given. (Received February 26, 1938.)

NEW PUBLICATIONS

PART I. PURE MATHEMATICS

ACKERMANN (W.). See HILBERT (D.).

ALBERT (A. A.). Modern higher algebra. (University of Chicago Science Series, edited by G. A. Bliss, H. H. Barrows, and Preston Kyes.) Chicago, University of Chicago Press, 1937. 14+319 pp.

BARROWS (H. H.). See ALBERT (A. A.).

BLISS (G. A.). See ALBERT (A. A.).

BOREL (E.). See MINEUR (H.), STOILOW (S.), and VOLTERRA (V.).

BOULIGAND (G.). Lignes de niveau. Lignes intégrales. Introduction à leur étude graphique. Edited by J. Devisme. Paris, Vuibert, 1937. 8+154 pp.

BOURION (G.). L'ultraconvergence dans les séries de Taylor. (Actualités Scientifiques et Industrielles, no. 472; exposés sur la théorie des fonctions, VIII.) Paris, Hermann, 1937. 47 pp.

BRITISH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE. Mathematical tables. Vol. 6: Bessel functions. Part I: Functions of orders zero and unity. Cambridge, University Press, 1937. 20+288 pp.

BROWN (B. H.). See BURTON (W. W.).

BURTON (W. W.). Analytic geometry. General editor, B. H. Brown. New York, Harcourt Brace, 1938. 11+262 pp.

CARTAN (E.). La théorie des groupes finis et continus et la géométrie différentielle traitées par la méthode du repère mobile. Leçons professées à la Sorbonne par Émile Borel. Edited by Jean Leray. (Cahiers Scientifiques, publiés sous la direction de Gaston Julia, no. 18.) Paris, Gauthier-Villars, 1937. 6+269 pp.

CUNNINGHAM (E.). See SYNGE (J. L.).

DEVISME (J.). See BOULIGAND (G.).

DOEHLEMANN (K.) and TIMERDING (H.). Projektive Geometrie. (Sammlung Göschen, no. 72.) Berlin and Leipzig, de Gruyter, 1937. 131 pp.

DOETSCH (G.). Theorie und Anwendung der Laplace-Transformation. Berlin, Springer, 1937. 16+436 pp.

ENGEL (F.). See LIE (S.).

FITE (W. B.). Advanced calculus. New York, Macmillan, 1938. 12+399 pp.

FORADORI (E.). Grundgedanken der Teiltheorie. Leipzig, Hirzel, 1937. 79 pp.

GALE (A. S.). See SMITH (P. F.).

GONTCHAROFF (W.). Détermination des fonctions entières par interpolation. (Actualités Scientifiques et Industrielles, no. 465; exposés sur la théorie des fonctions, VII.) Paris, Hermann, 1937. 49 pp.

HAMEL (G.). Integralgleichungen. Einführung in Lehre und Gebrauch. Berlin, Springer, 1937. 8+166 pp.

HARDY (G. H.). See SYNGE (J. L.).

HEEGAARD (P.). See LIE (S.).

HILBERT (D.) and ACKERMANN (W.). Grundzüge der theoretischen Logik. 2d edition. Berlin, Springer, 1938. 8+133 pp.

HOSTINSKY (B.). See VOLTERRA (V.).

JULIA (G.). See CARTAN (E.).

KARAMATA (J.). Sur les théorèmes inverses des procédés de sommabilité. (Actualités

Scientifiques et Industrielles, no. 450; exposés sur la théorie des fonctions, VI.) Paris, Hermann, 1937. 47 pp.

KAVAN (G.). Factor tables giving the complete decomposition into prime factors of all numbers up to 256,000. London, Macmillan, 1937. 12+514 pp.

KYES (P.). See ALBERT (A. A.).

LAMBERT (R.). Structure générale des nomogrammes des systemes nomographiques. With a preface by M. d'Ocagne. (Actualités Scientifiques et Industrielles, no. 493; exposés sur les procédés généraux de calcul, I.) Paris, Hermann, 1937. 64 pp.

LERAY (J.). See CARTAN (E.).

LEVI (B.). Analisi matematica, algebrica ed infinitesimale. Bologna, Zanichelli, 1937. 7+541 pp.

LIE (S.). Gesammelte Abhandlungen. Edited by Friedrich Engel and Poul Heegaard. Vol. 2: Geometrische Abhandlungen. Part II. 10+332 pp. Anmerkungen zum zweiten Bande. 174 pp. Leipzig, Teubner; Oslo, Aschehoug, 1937.

MAHAJANI (G. S.). The application of moving axes methods to the geometry of curves and surfaces. Poona, Aryabhushan Press, 1937. 7+60 pp.

MALENGREAU (J.). Essai sur les fondements de la géométrie euclidienne. Lausanne, Payot, 1938. 311 pp.

MARCH (H. W.) and WOLFF (H. C.). Calculus. 3d edition. (Modern Mathematical Texts, edited by C. S. Slichter.) New York and London, McGraw-Hill, 1937. 17+424 pp.

MINEUR (H.). Technique de la méthode des moindres carrés. (Monographies des Probabilités, publiés sous la direction de E. Borel, no. 2.) Paris, Gauthier-Villars, 1938. 8+93 pp.

MORRILL (W. K.). Plane trigonometry. New York, Farrar and Rinehart, 1938. 9+167 pp.

NEELLEY (J. H.). See SMITH (P. F.).

D'OCAGNE (M.). See LAMBERT (R.).

RUNNING (T. R.). Graphical calculus. Ann Arbor, Wahr. 1937. 82 pp.

RUSSELL (B.). The principles of mathematics. 2d edition. London, Allen and Unwin, 1937. 39+534 pp.

SLICHTER (C. S.). See MARCH (H. W.).

SMITH (P. F.), GALE (A. S.), and NEELLEY (J. H.). New analytic geometry. Alternate edition. Boston, Ginn, 1938. 10+336 pp.

STOILOW (S.). Leçons sur les principes topologiques de la théorie des fonctions analytiques. (Collection de Monographies sur la Théorie des Fonctions, publiés sous la direction de E. Borel.) Paris, Gauthier-Villars, 1938. 10+148 pp.

SYNGE (J. L.). Geometrical optics. An introduction to Hamilton's method. (Cambridge Tracts in Mathematics and Mathematical Physics, edited by G. H. Hardy and E. Cunningham, no. 37.) Cambridge, University Press; New York, Macmillan, 1937. 9+110 pp.

TIMERDING (H.). See DOEHLEMANN (K.).

VOLTERRA (V.) and HOSTINSKY (B.). Opérations infinitésimales linéaires. Applications aux équations différentielles et fonctionelles. (Collections de Monographies sur la Théorie des Fonctions, publiés sous la direction de E. Borel.) Paris, Gauthier-Villars, 1938. 7+238 pp.

WOLFF (H. C.). See MARCH (H. W.).

PART II. APPLIED MATHEMATICS

BOEHM (C.), LORENZ (P.), and STANISZEWSKI (J.). Umwandlung von Lebensversicherung. (Versicherungsmathematische Aufgabensammlung, published by the Deutschen Aktuarverein, no. 2.) Leipzig and Berlin, Teubner, 1937. 14+52 pp.

BOEHM (C.) and ROSE (E.). Beiträge und Deckungsrucklagen in der Lebensversicherung. (Versicherungsmathematische Aufgabensammlung, published by the Deutschen Aktuarverein, no. 1.) Leipzig and Berlin, Teubner, 1937. 11+75 pp.

BOUCHÉ (C.). Kolbenverdichter: Einführung in Arbeitsweise und Bau von Luft- und Gasverdichten mit Kolbenbewegung. Berlin, Springer, 1937. 4+125 pp.

BOULIGAND (G.). Précis de mécanique rationnelle. Edited by J. Dollon. 2d edition, revised and augmented. Paris, Vuibert, 1937. 8+341 pp.

BRÜCHE (E.). See BUSCH (H.).

BURSTYN (W.). Elektrische Kontakte: Grundlagen für den Pratiker. Berlin, Springer, 1937. 7+79 pp.

BUSCH (H.) and BRÜCHE (E.), Edited by. Beiträge zur Elektronenoptik. Leipzig, Barth, 1937. 156 pp.

COURANT (R.) and HILBERT (D.). Methoden der mathematischen Physik. Vol. 2. Berlin, Springer, 1937. 16+549 pp.

DEUTSCHEN AKTUARVEREIN. See BOEHM (C.).

DOLLON (J.). See BOULIGAND (G.).

ECK (B.). Ventilatoren: Entwurf und Betrieb der Schleuder- und Schraubengebläse. Berlin, Springer, 1937. 7+197 pp.

ERIKSON (H. A.). See ZELENY (A.).

FLOYD (W. F.). See WOODGER (J. H.).

GIBRAT (R.). See TINBERGEN (J.).

GOEBEL (O.). Das Wirtschaftsganze im Blickfeld des Ingenieurs: eine Einführung in die Volkswirtschaft. (Schriftenreihe Ingenieurfortbildung, edited by G. V. Hanffstengel, no. 3.) Berlin, Springer, 1937. 5+111 pp.

GUILLAUME (G.) and (E.). Économique rationnelle. No. 1: Méthode. L'économique élevée au rang d'une science exacte fondée axiomatiquement. 44 pp. No. 2: L'économie pure. Le domaine des mécanismes élémentaires. Formation des prix et des salaires. 76 pp. No. 3: L'économie pure en interférence avec le domaine juridique. Mécanisme de la formation des bénéfices, des déficiences du pouvoir d'achat et des crises. 55 pp. No. 4: Le domaine économique-juridique national en interférence avec des mondes étrangers. 59 pp. No. 5: Modèles mathématiques des mondes économiques. 136 pp. (Actualités Scientifiques et Industrielles, nos. 504–508; exposés publiés sous la direction de G. and E. Guillaume.) Paris, Hermann, 1937.

GUMBEL (E.-J.). La durée extrême de la vie humaine. (Actualités Scientifiques et Industrielles, no. 520; Statistiques Mathématiques, II.) Paris, Hermann, 1937. 65 pp.

HANFFSTENGEL (G. V.). See GOEBEL (O.).

HILBERT (D.). See COURANT (R.).

HUMBERT (P.). De Mercure à Pluton, planètes et satellites. Paris, Michel, 1937. 189 pp.

KEMBLE (E.C.). Fundamental principles of quantum mechanics. New York, McGraw-Hill, 1937. 18+611 pp.

LORENZ (P.). See BOEHM (C.).

MIHOC (G.). See ONICESCU (O.).

ONICESCU (O.) and MIHOC (G.). La dépendance statistique. Chaînes et familles de chaînes discontinues. (Actualités Scientifiques et Industrielles, no. 503; exposés d'analyse générale, VII.) Paris, Hermann, 1937. 47 pp.

REGLER (F.). Grundzüge der Röntgenphysik. (Sonderbände zur Strahlentherapie, vol. 21.) Berlin and Vienna, Urban and Schwarzenberg, 1937. 467 pp.

RIDER (P. R.). Essentials of the mathematics of investment. New York, Farrar and Rinehart, 1938. 10+162 pp.

ROSE (E.). See BOEHM (C.).

SCHAEFER (C.). Einführung in die theoretische Physik. Vol. 3, part 2: Quantentheorie. Berlin and Leipzig, de Gruyter, 1937. 7+510 pp.

SCHLEUSNER (A.). Strenge Theorie der Knickung und Biegung. Leipzig, Teubner, 1937. 144 pp.

SNEDECOR (G. W.). Statistical methods. Applied to experiments in agriculture and biology. Ames, Iowa, Collegiate Press, 1937. 13+341 pp.

STANISZEWSKI (J.). See BOEHM (C.).

TARSKI (A.). See WOODGER (J. H.).

THIBAUD (J.). Vie et transmutation des atomes. (Sciences d'aujourd'hui.) Paris, Michel, 1937. 236 pp.

TINBERGEN, (J.). An econometric approach to business cycle problems. (Actualités Scientifiques et Industrielles, no. 525; Impassés Economiques, exposés publiés sous la direction de Robert Gibrat, II.) Paris, Hermann, 1937. 75 pp.

WAUGH (A. B.). Elements of statistical method. New York and London, McGraw-Hill, 1938. 15+381 pp.

WOODGER (J. H.). The axiomatic method in biology. With appendices by Alfred Tarski and W. F. Floyd. Cambridge, University Press; New York, Macmillan, 1937. 10+174 pp.

ZELENY (A.) and ERIKSON (H. A.). A manual of physical measurements. 6th edition. New York and London, McGraw-Hill, 1937. 23+273 pp.